Analog quantum algorithms for combinatorial optimization on neutral-atom devices: from graph embeddings to tunneling in quantum annealing

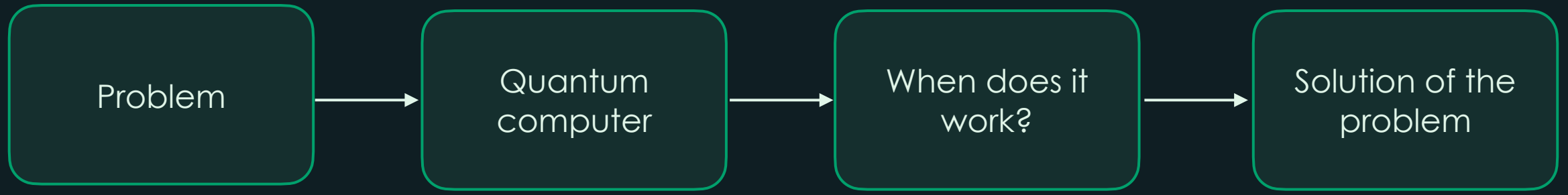
Elie BERMOT

PhD Defense *viva*
12 October 2026

Supervised by Dr Simon Apers – IRIF, Dr Louis-Paul Henry - PASQAL and formally Dr Frédéric Magniez - IRIF

Outline of this thesis

When does a neutral-atom quantum computer help solve a hard optimization problem and how do we make it happen?



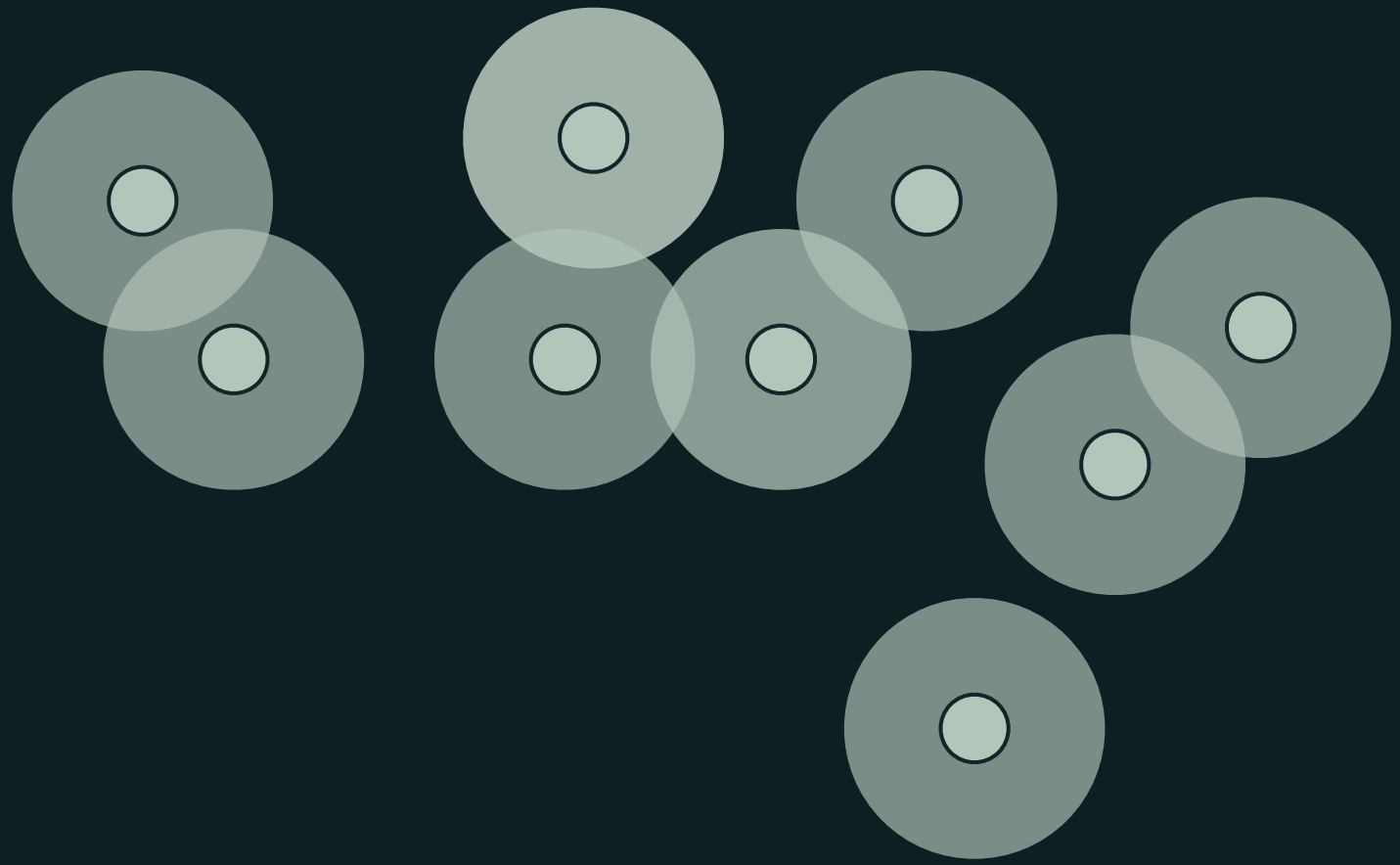
Pasqal Defining the quantum reality

Motivation & Background

From everyday puzzles to quantum computers

Optimization problems are everywhere

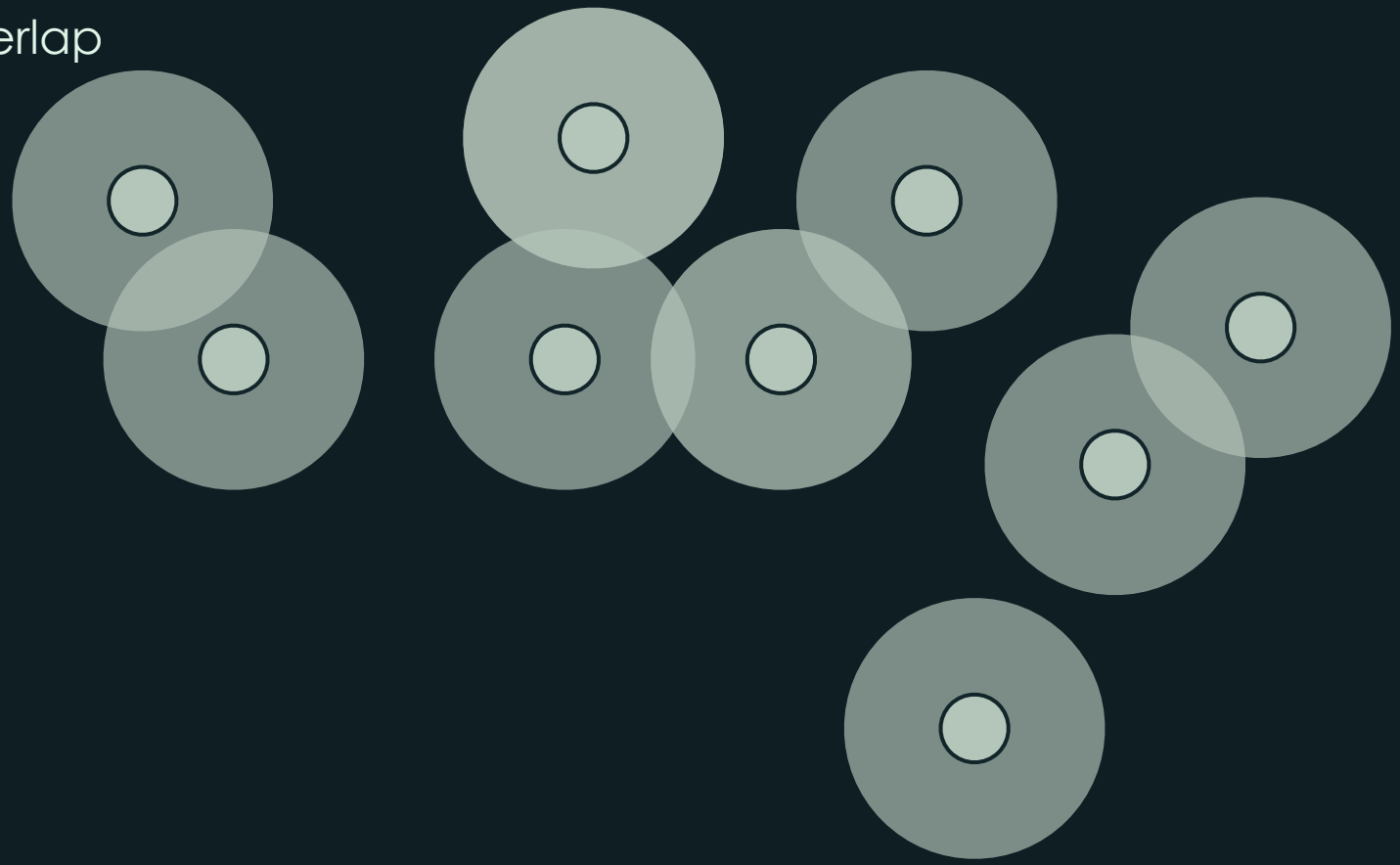
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Problem: Which dancers can spin at the same time without invading a neighbor's space?

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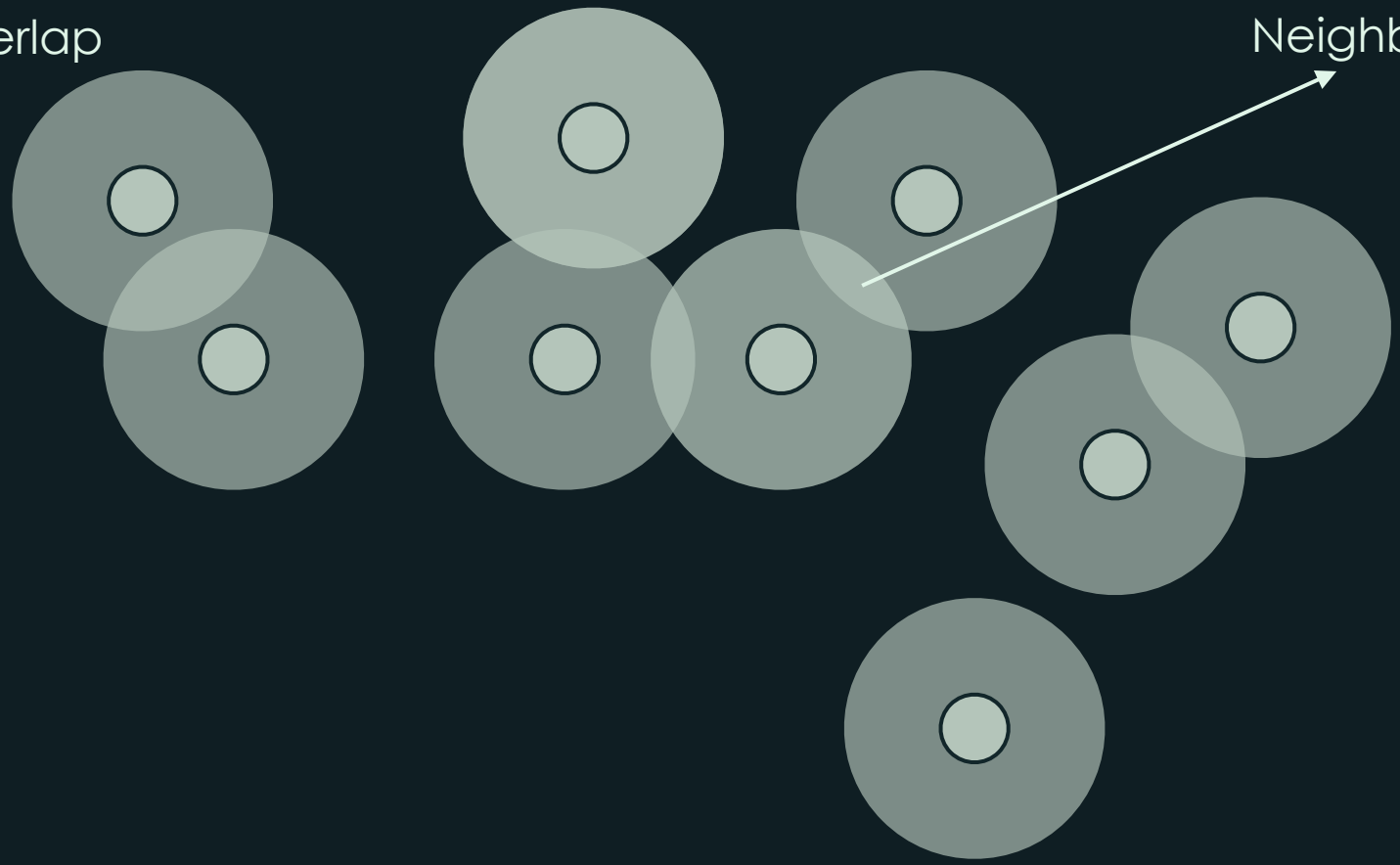
Neighbors=two disks overlap



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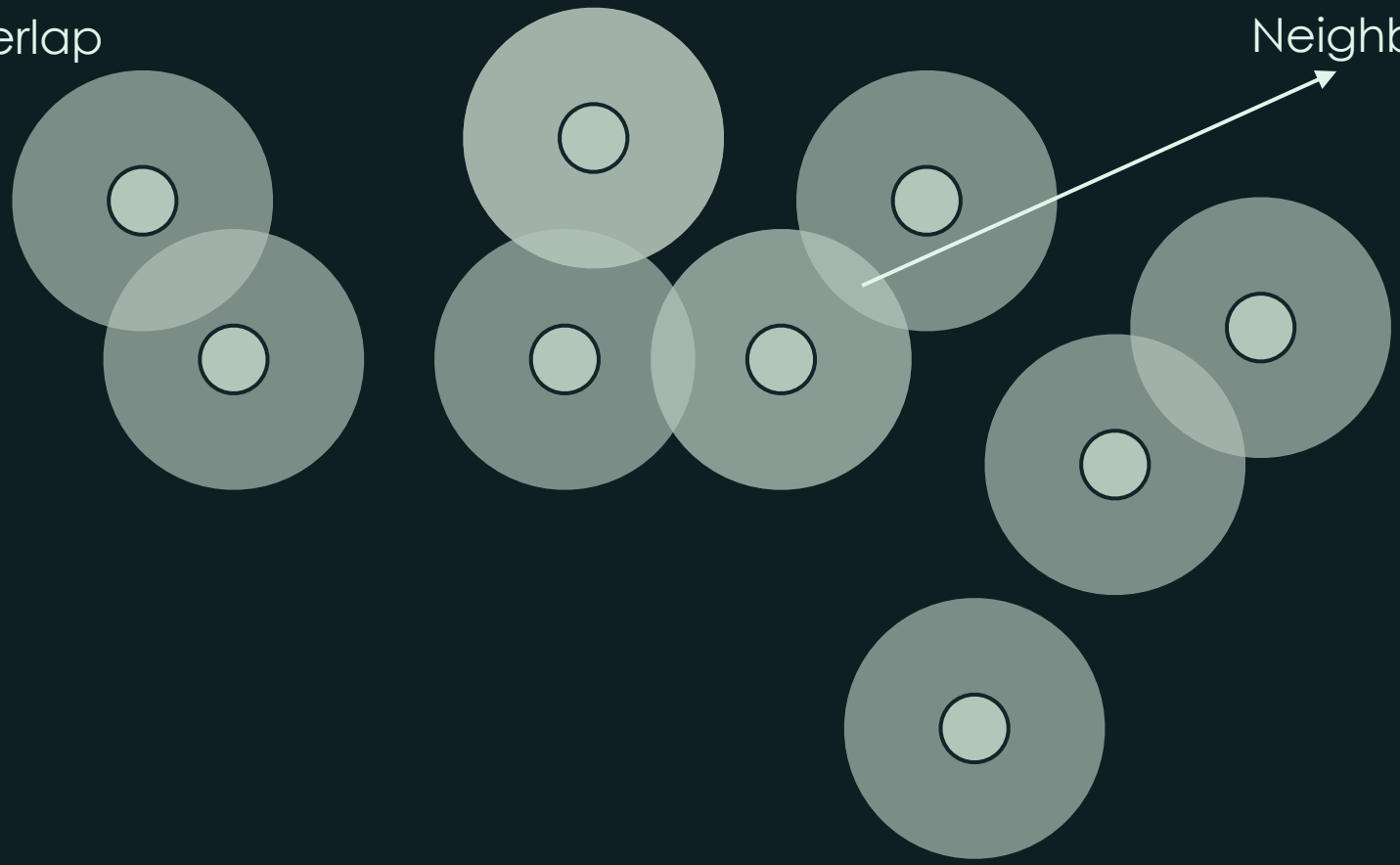
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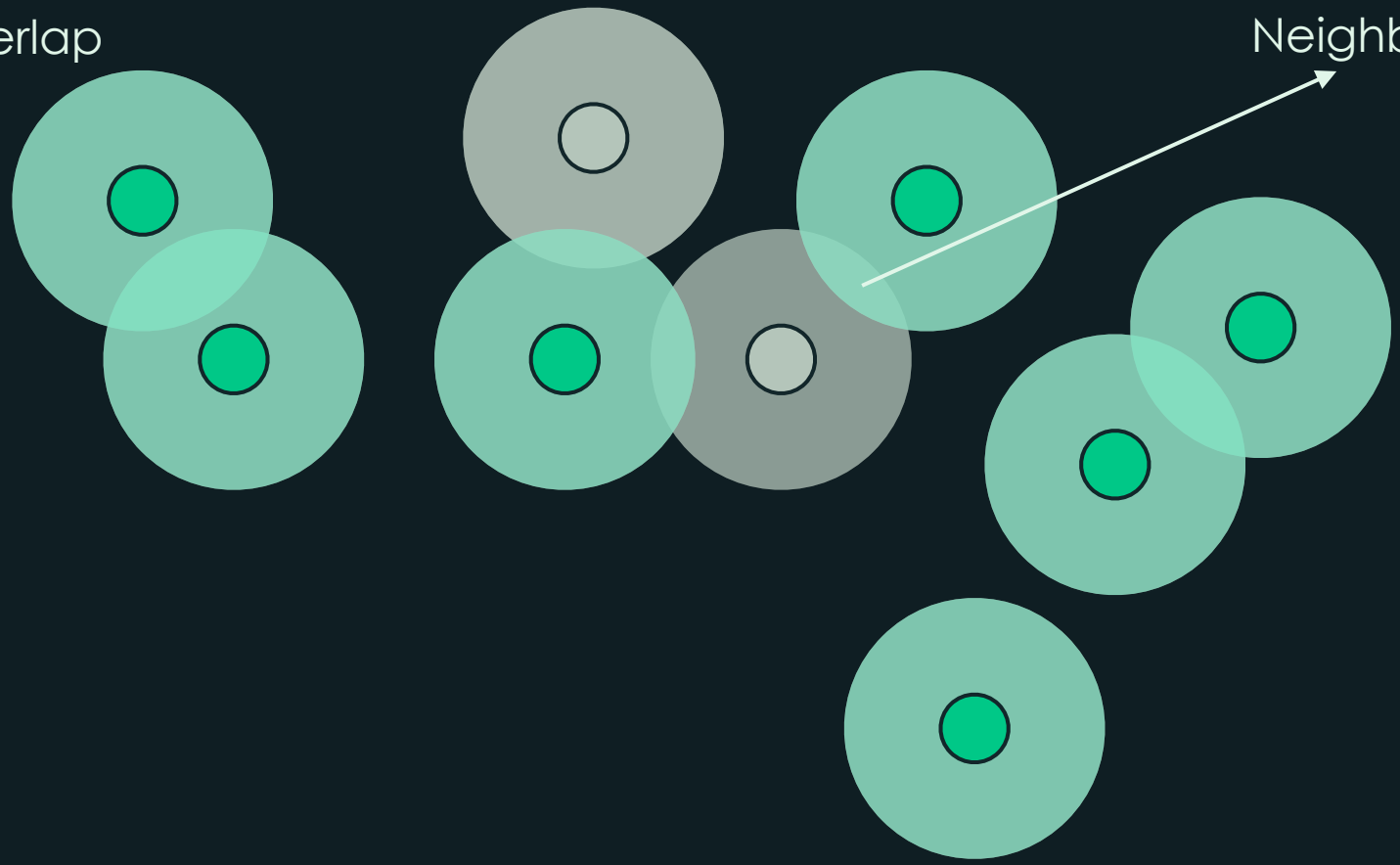


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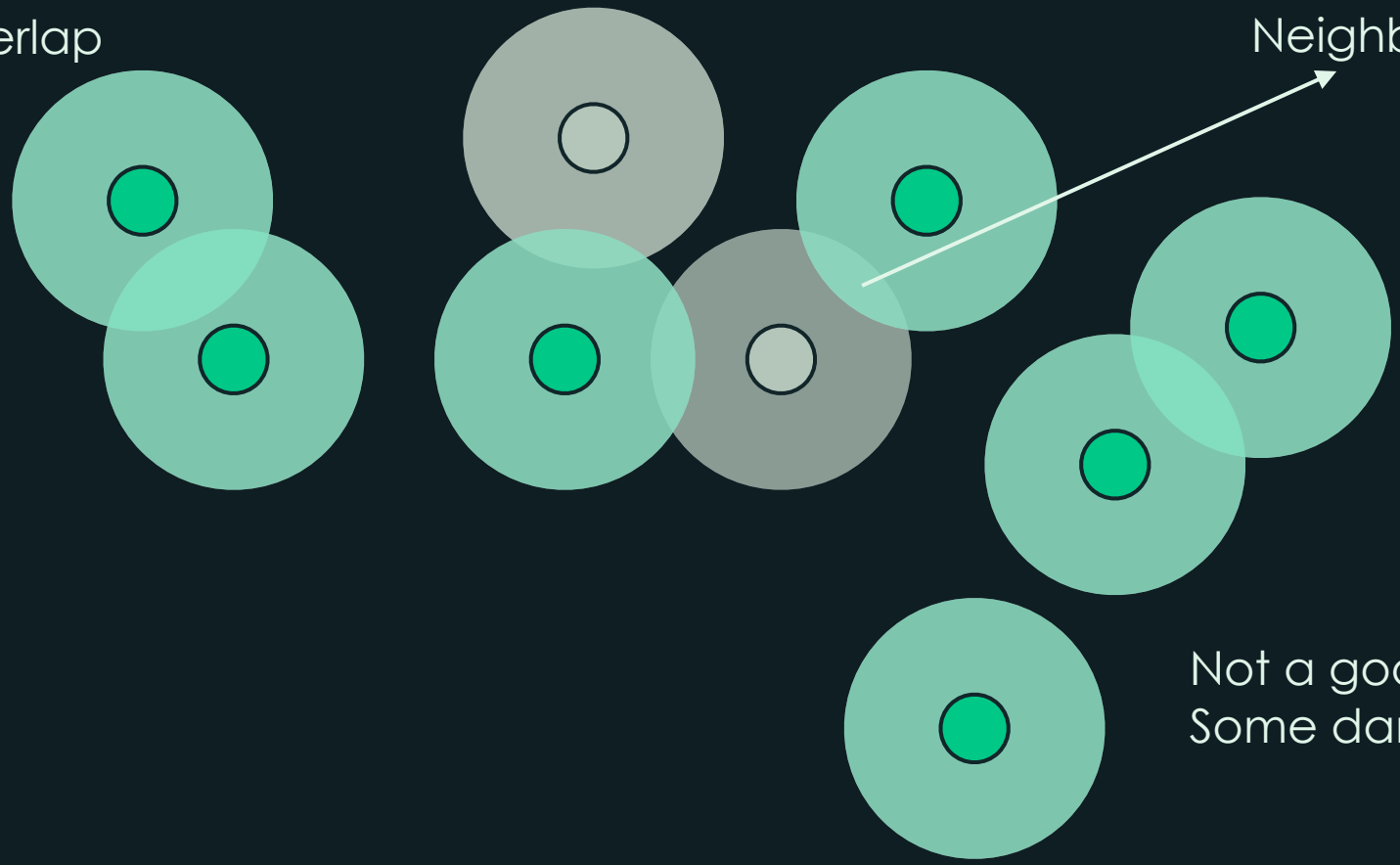


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Neighbors cannot both spin

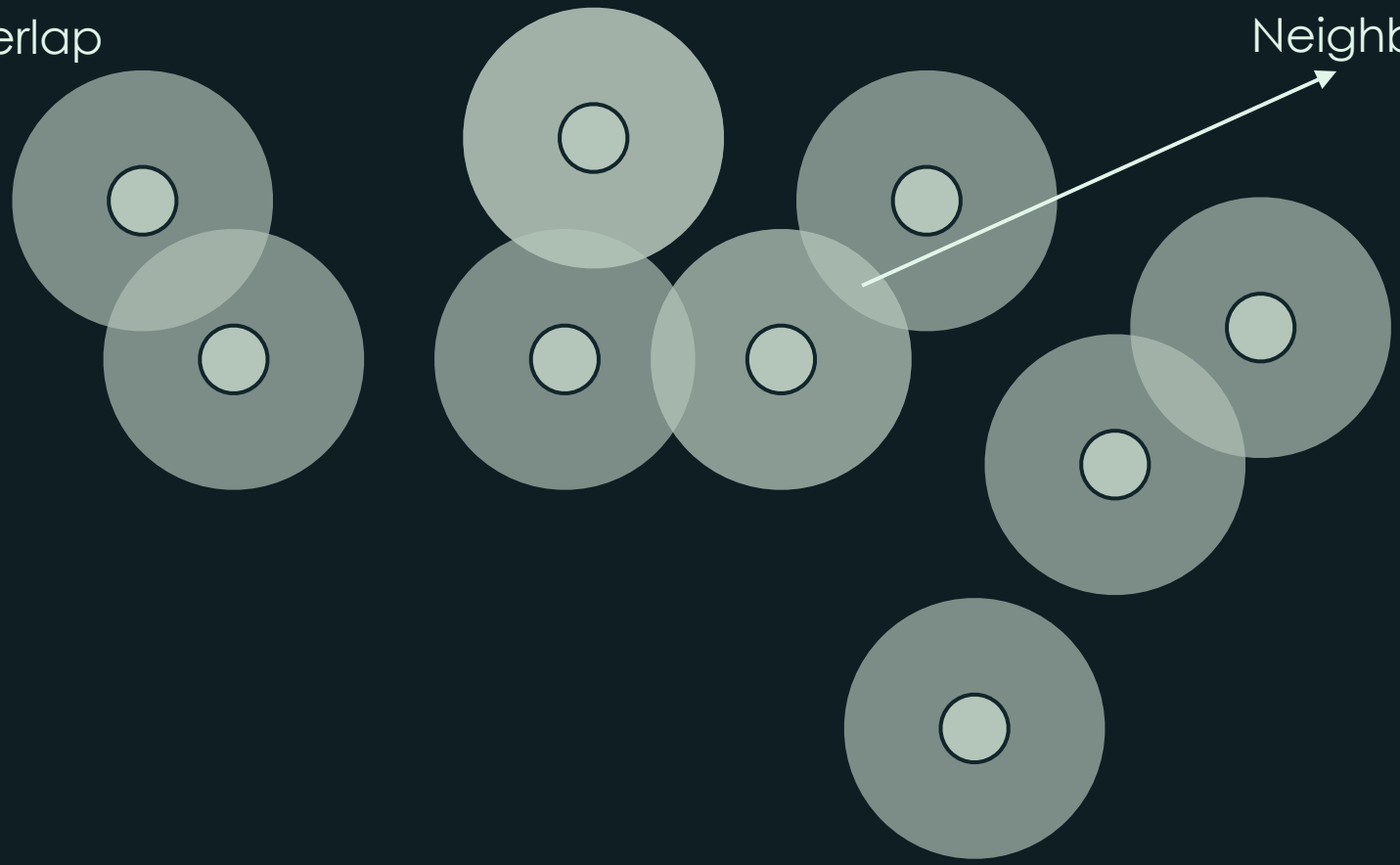
Not a good configuration
Some dancers hit each other!

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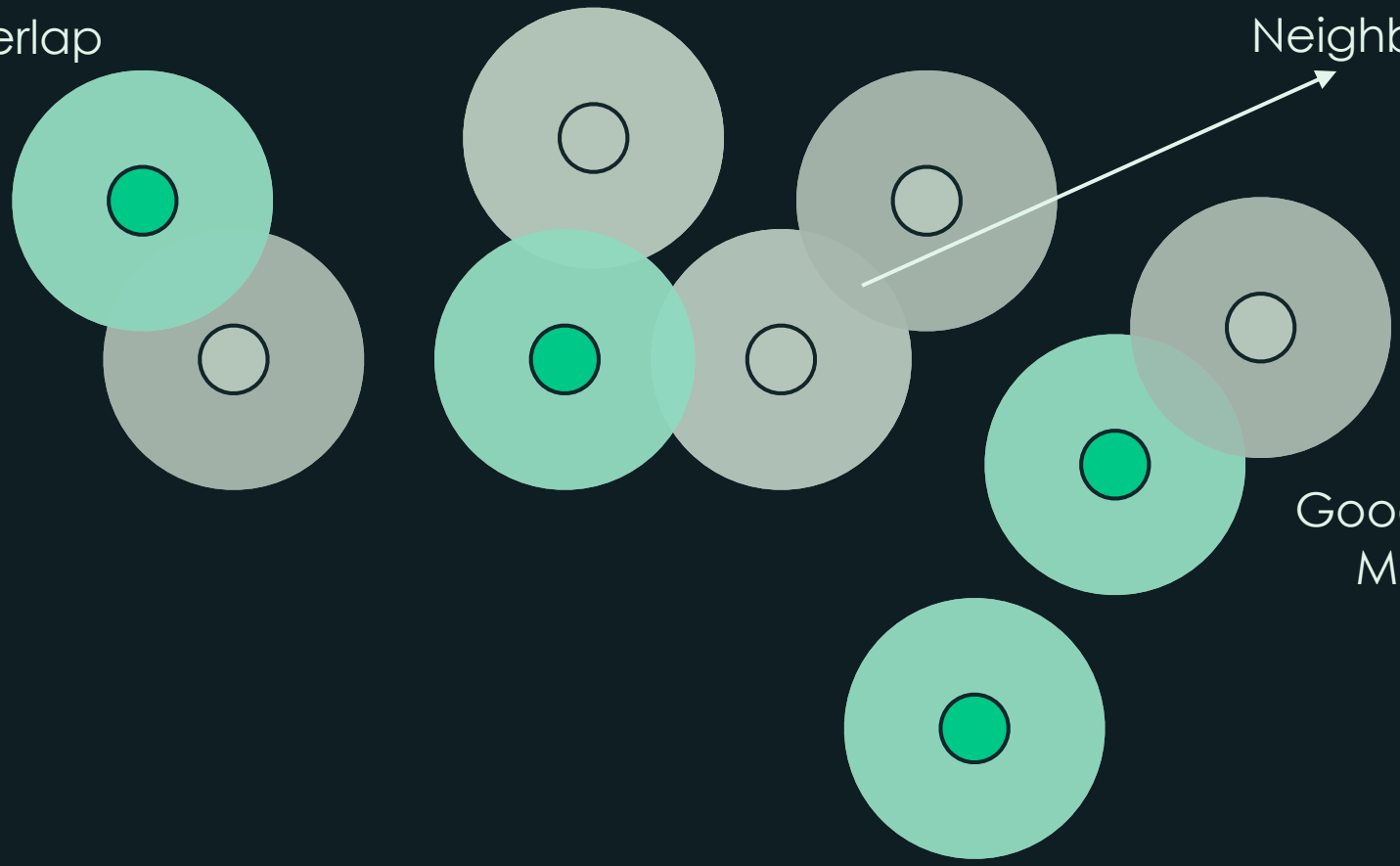


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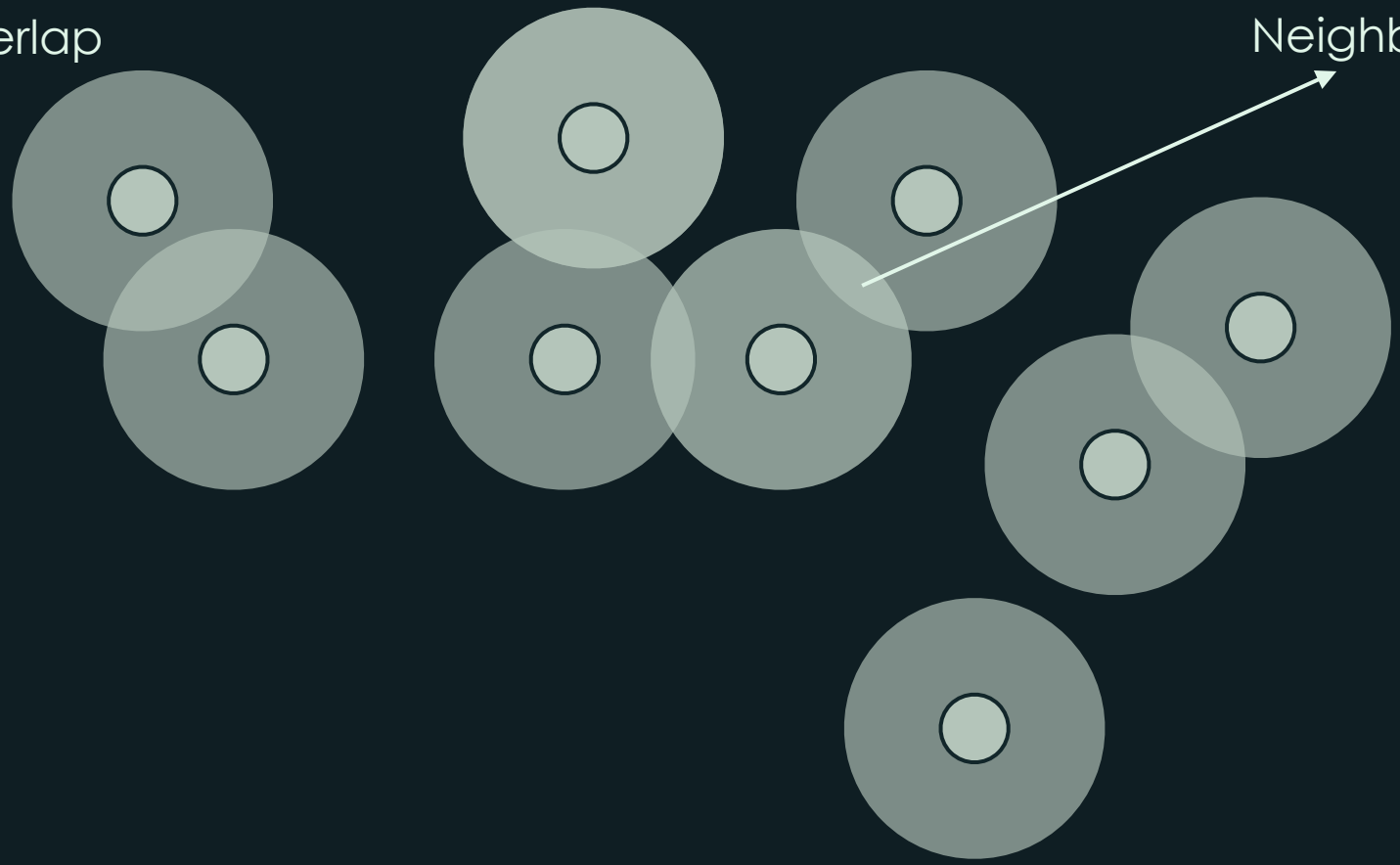
Good configuration
More dancers!

Problem: Which dancers can spin at the same time without invading a neighbor's space?

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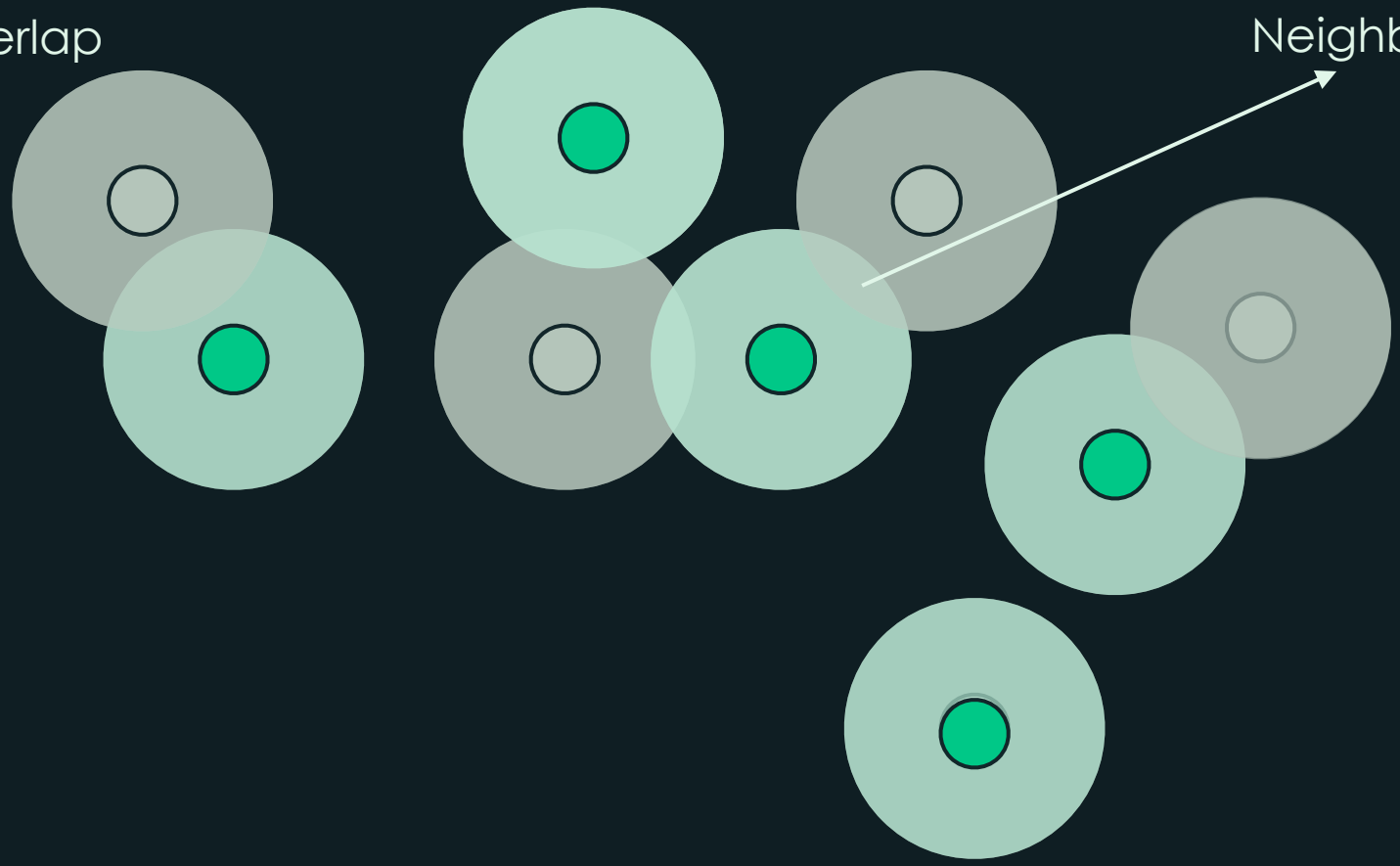


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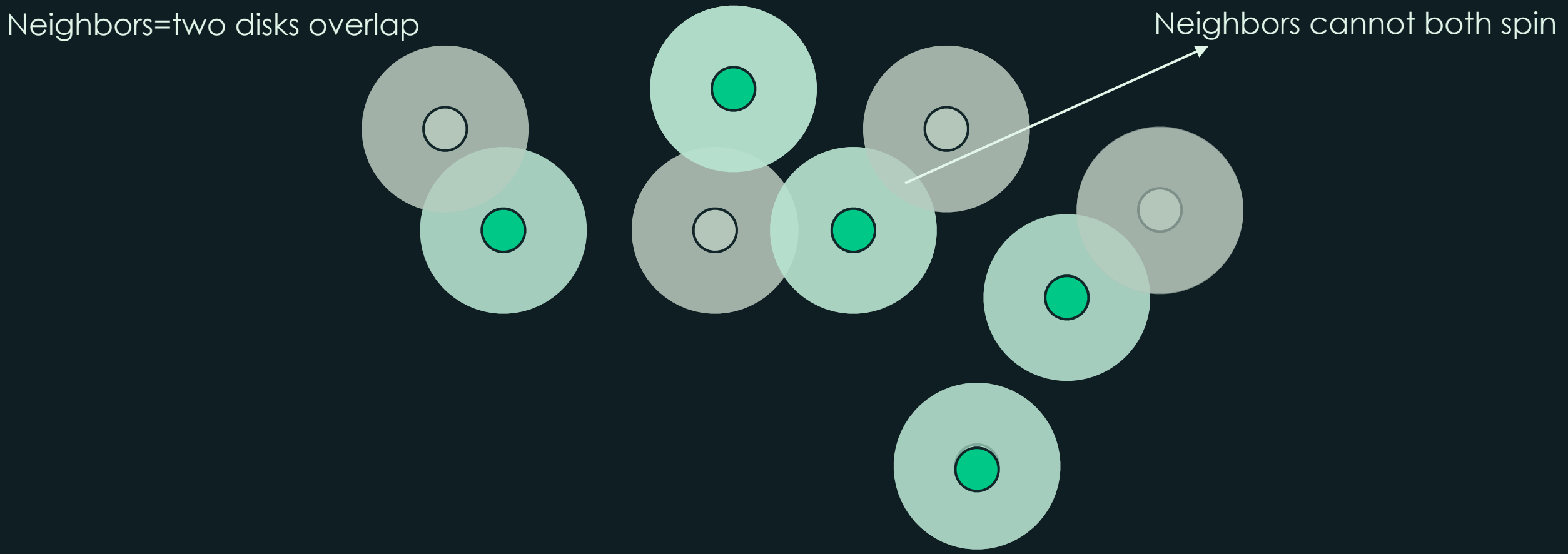


Neighbors cannot both spin

Problem: Which dancers can spin at the same time without invading a neighbor's space?

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The green circles correspond to a good configuration, example of the solution of what is called a maximum independent set (MIS)

Optimization problems are everywhere

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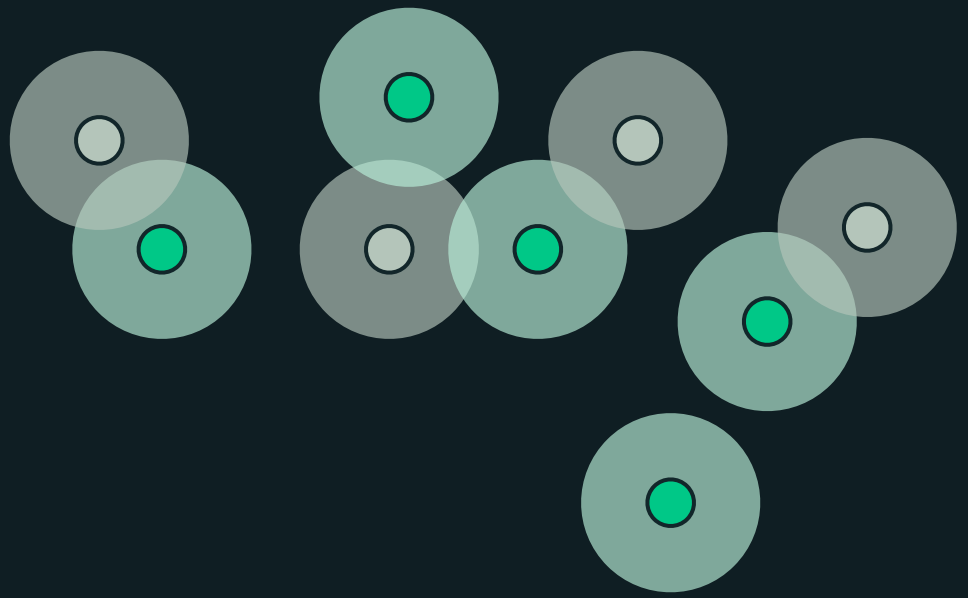
Independent set: a set of dancers with no two neighbors
maximum independent set: largest independent set

Problem: Which dancers can spin at the same time without invading a neighbor's space?

The green circles correspond to a good configuration, example of the solution of what is called a maximum independent set (MIS)

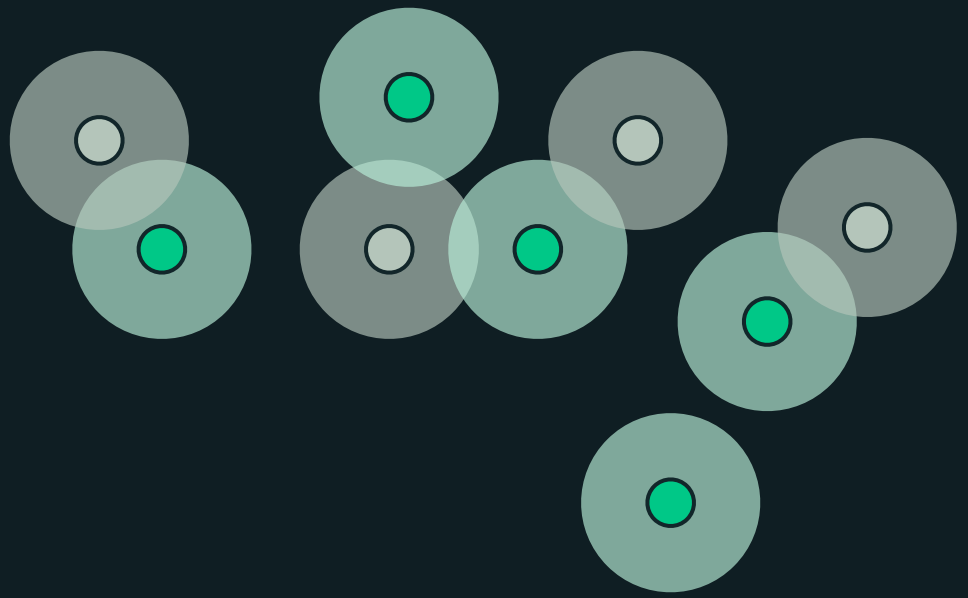
Hardness of solving such problems

How hard is it to find the MIS?



Hardness of solving such problems

How hard is it to find the MIS?



20 dancers → $2^{20} = 1048576$

300 dancers → $2^{300} \approx 10^{90}$

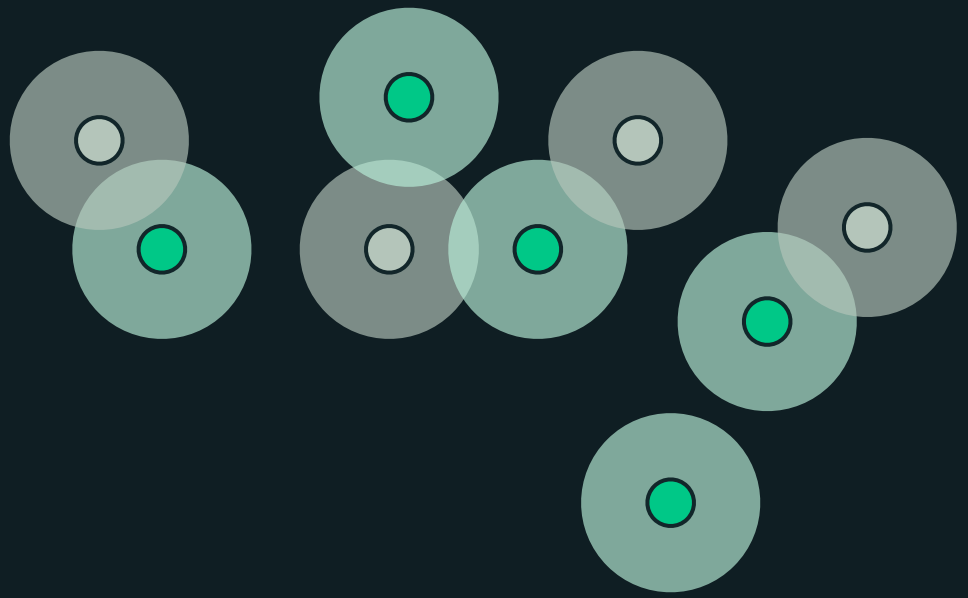
n dancers → 2^n configurations: exponential number of options

→ naive approach: explore all of them!

→ NP-hard problem: no polynomial algorithm is known.

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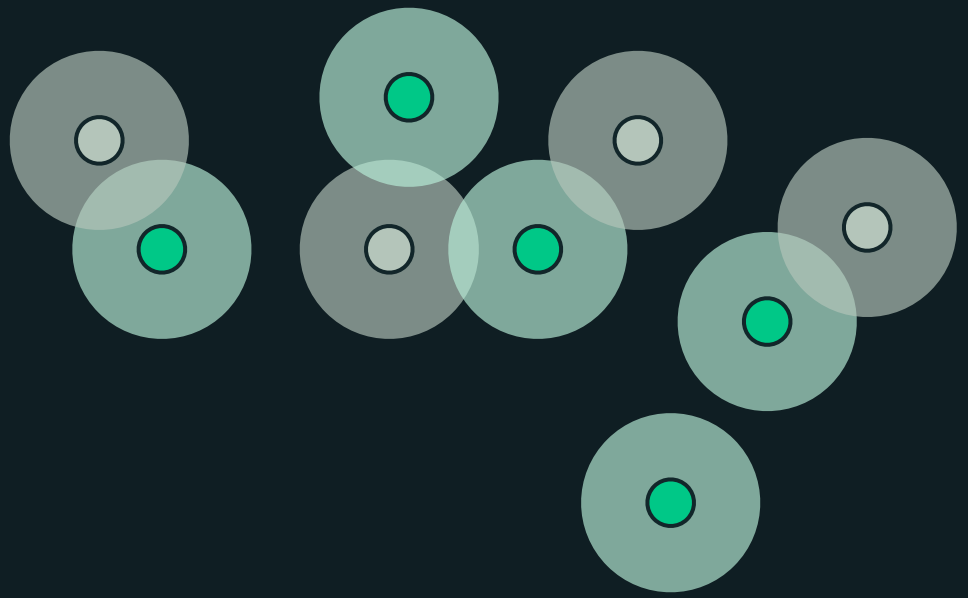
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→ NP-hard problem: no polynomial algorithm is known.

One approach: Approximation algorithms to estimate the solution of this problem:

➡ Simulated annealing

➡ Quantum methods

➡ …

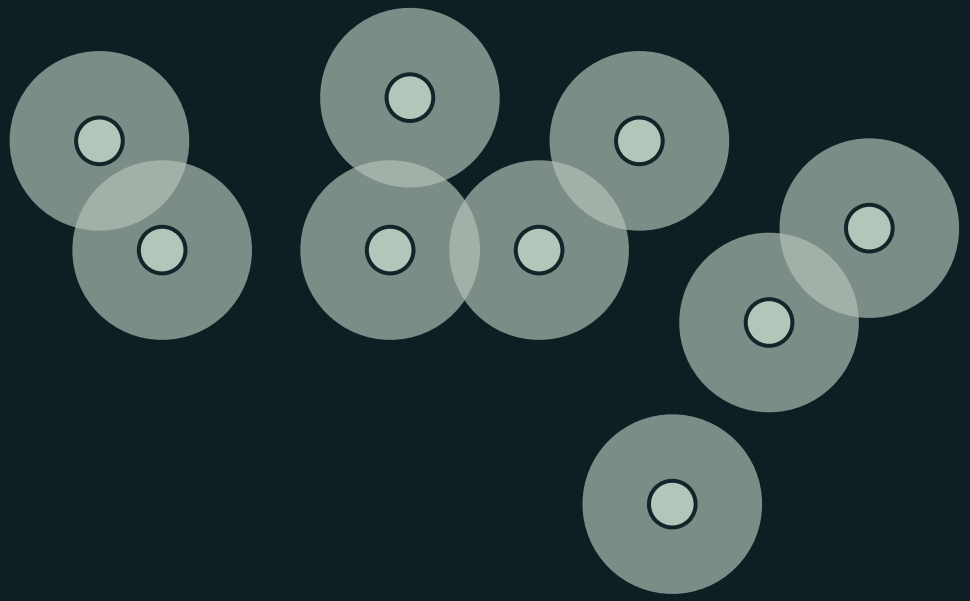
Unit disk (UD) MIS

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Dancers → vertices
Overlap → edges

Unit disk (UD) MIS

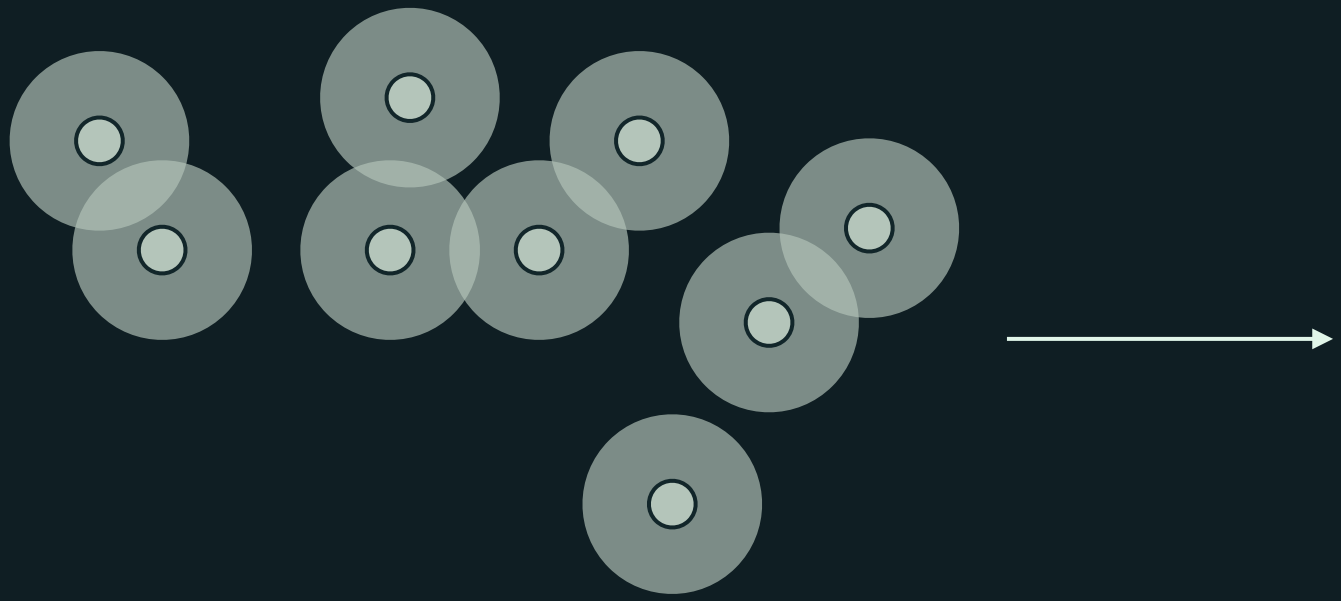
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Dancer

Unit disk (UD) MIS

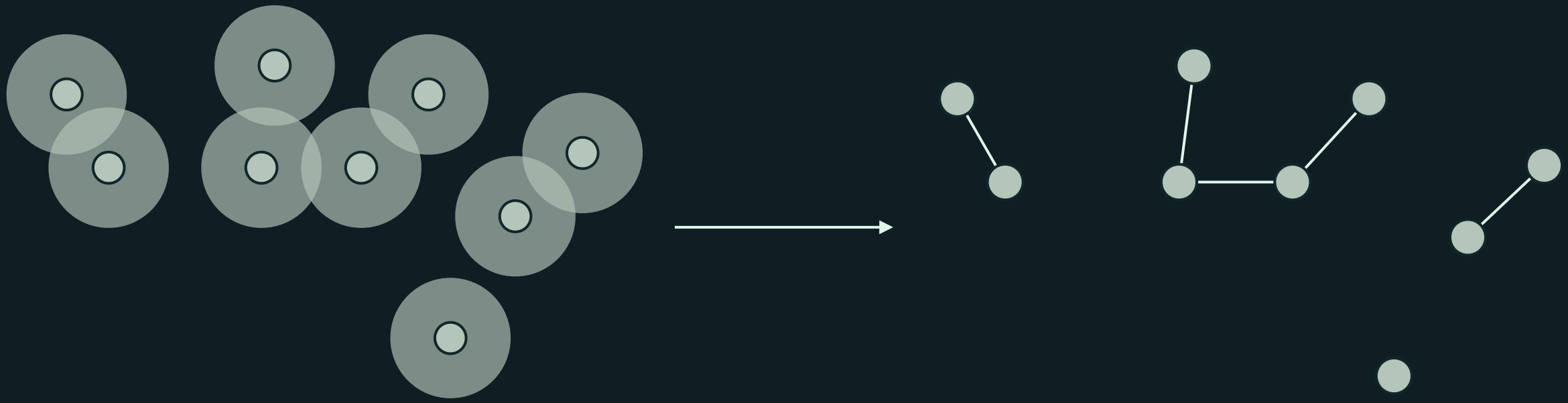
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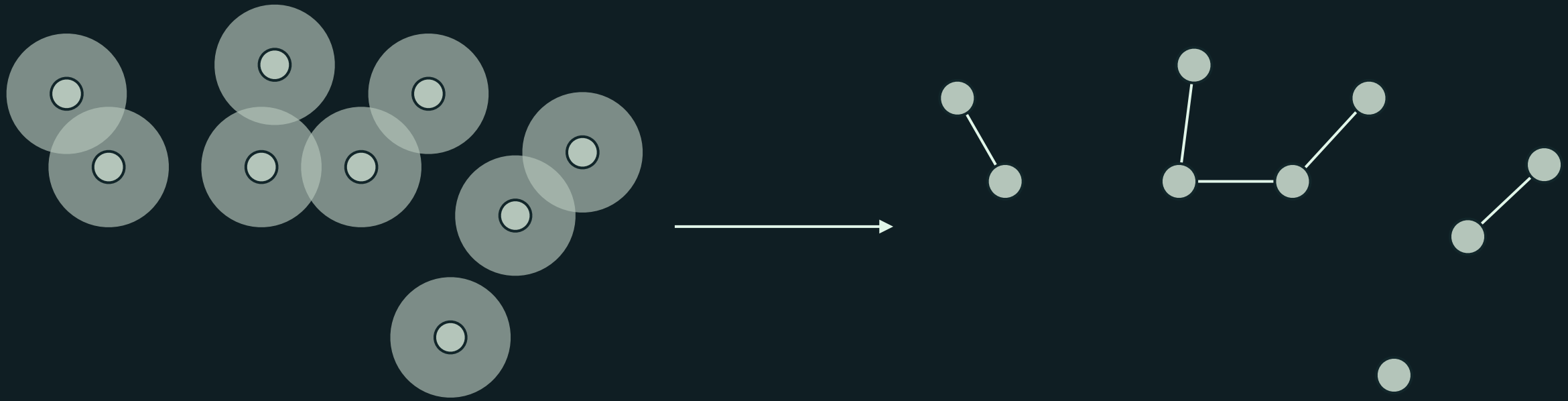
Dancer

Graph

Unit disk (UD) MIS

Dancers → vertices
Overlap → edges

Independent set: no link between vertices of the set



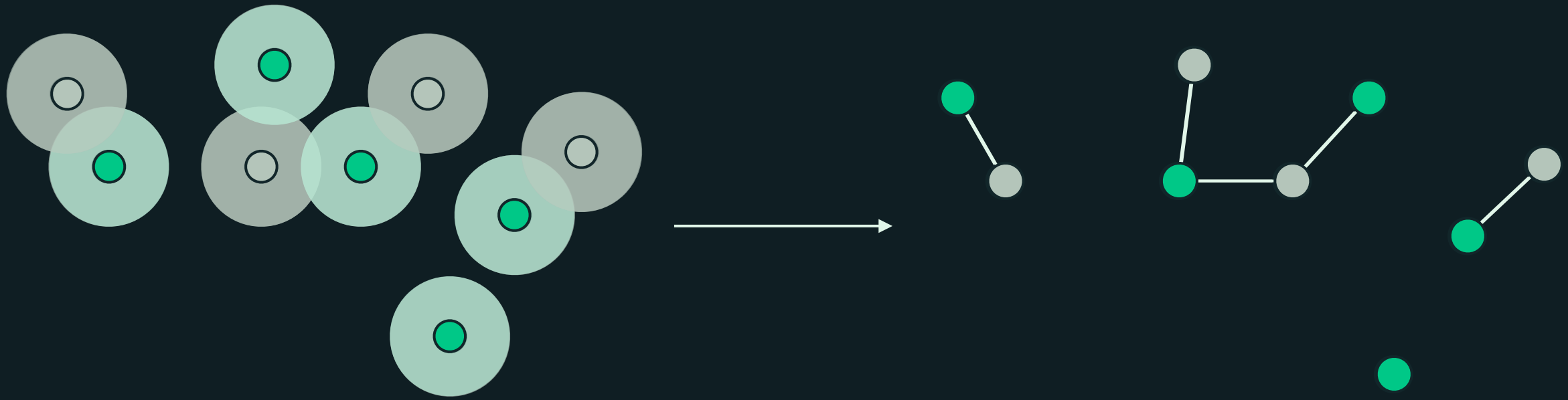
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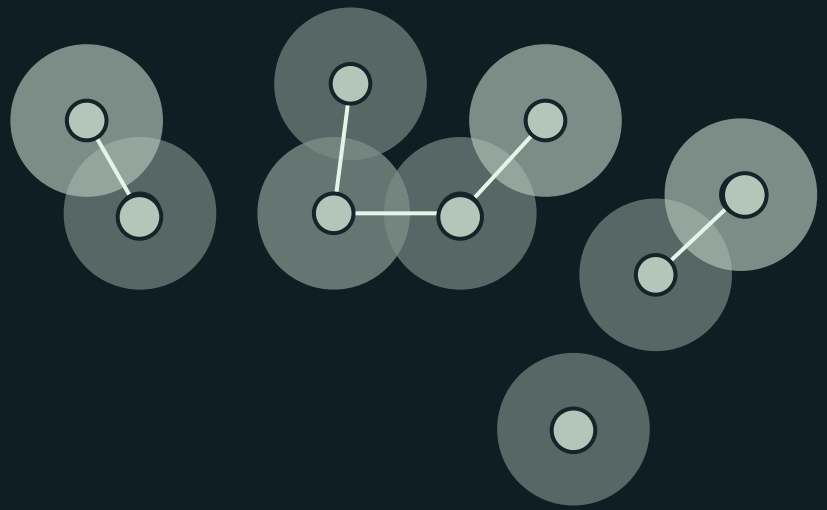
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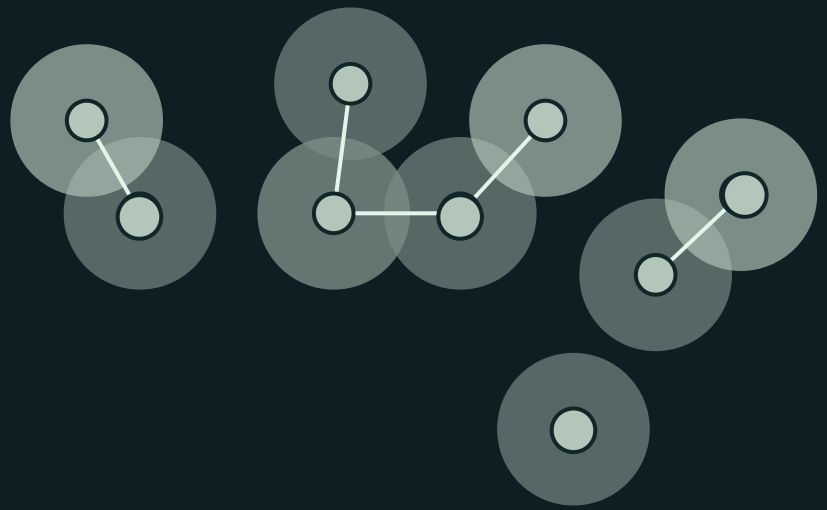
Finding the MIS: NP-hard

From a graph to a cost function

From a graph to a cost function

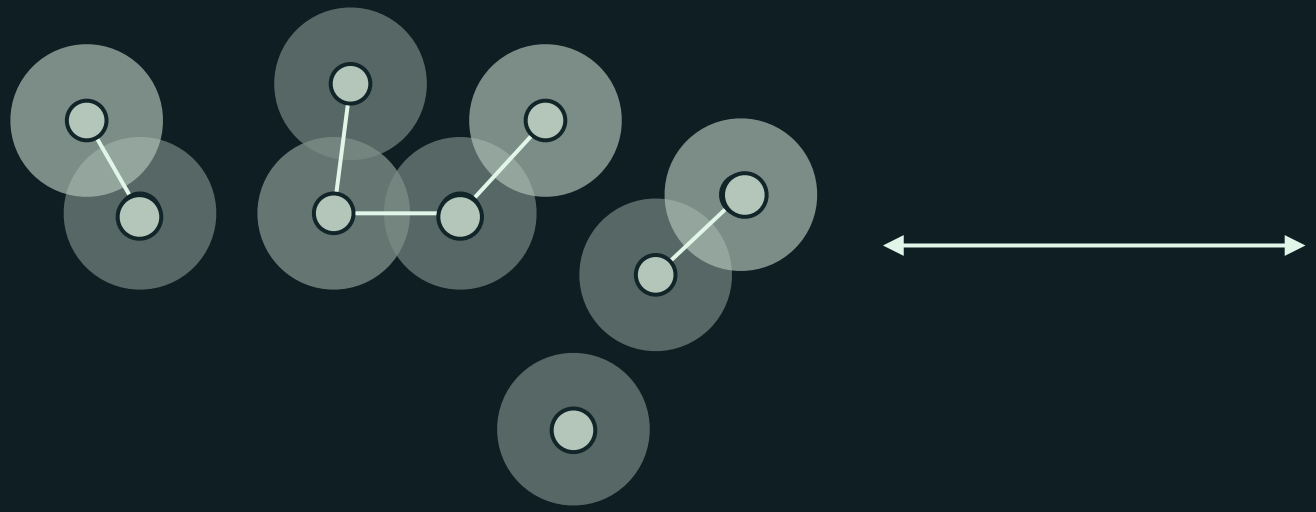


From a graph to a cost function



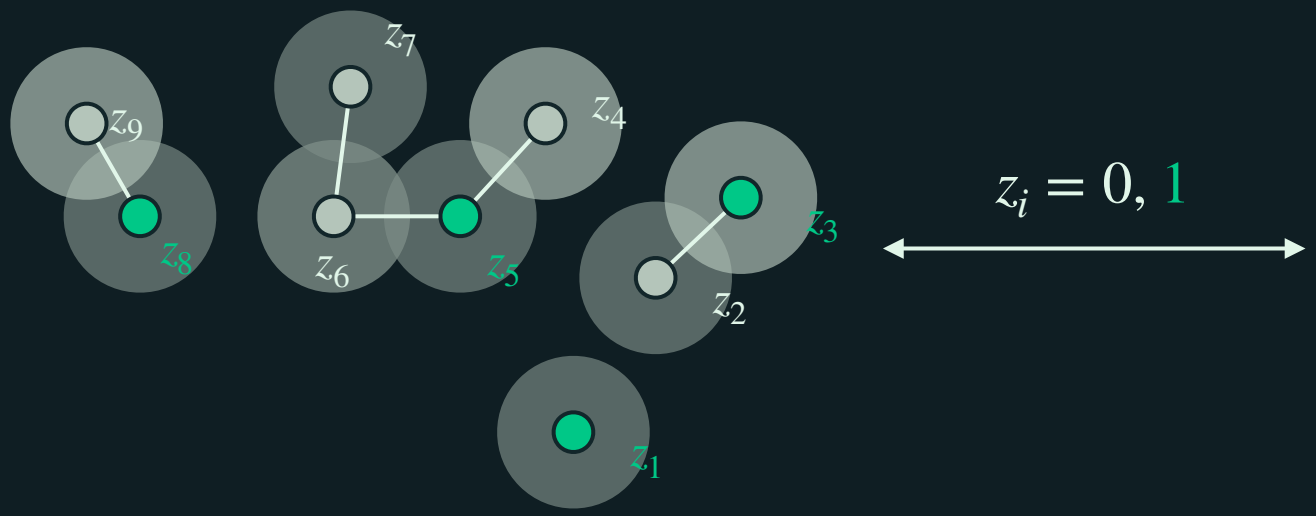
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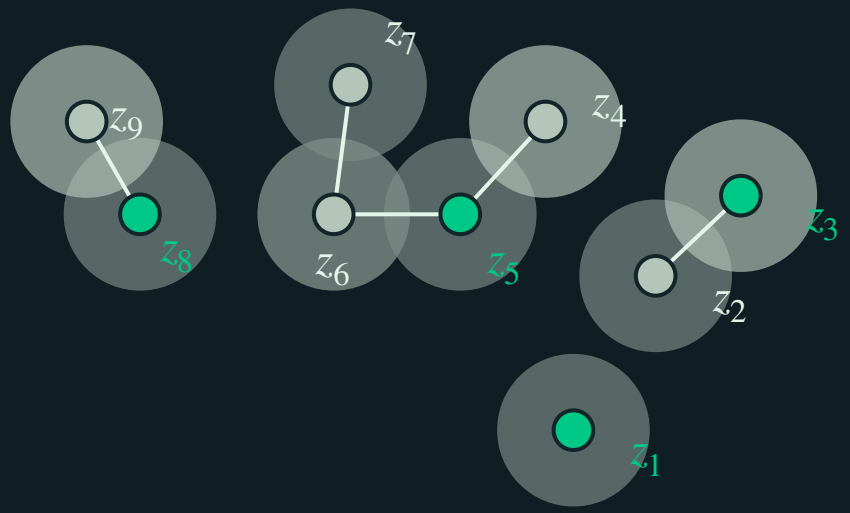
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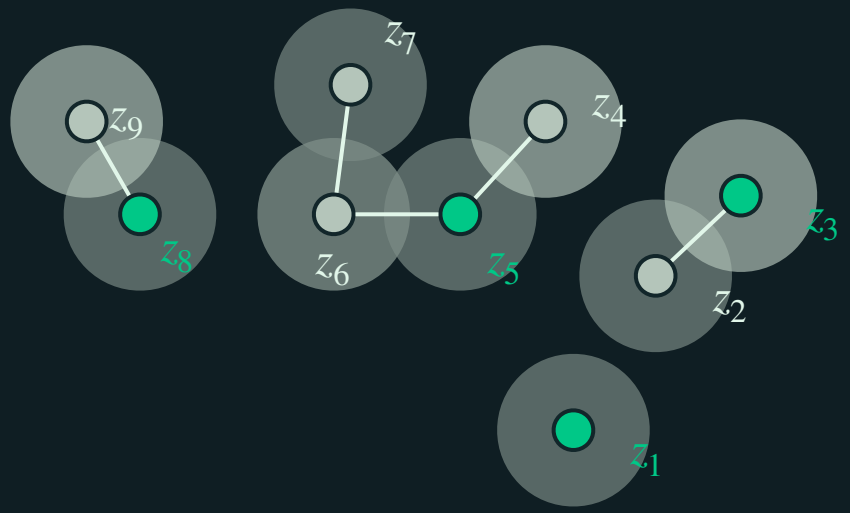
$z_i = 0, 1$

Associated quadratic cost function

$$C(z_1, \ldots, z_{|V|}) = -\sum_{i \in V} z_i + U \sum_{(i,j) \in E} z_i z_j$$

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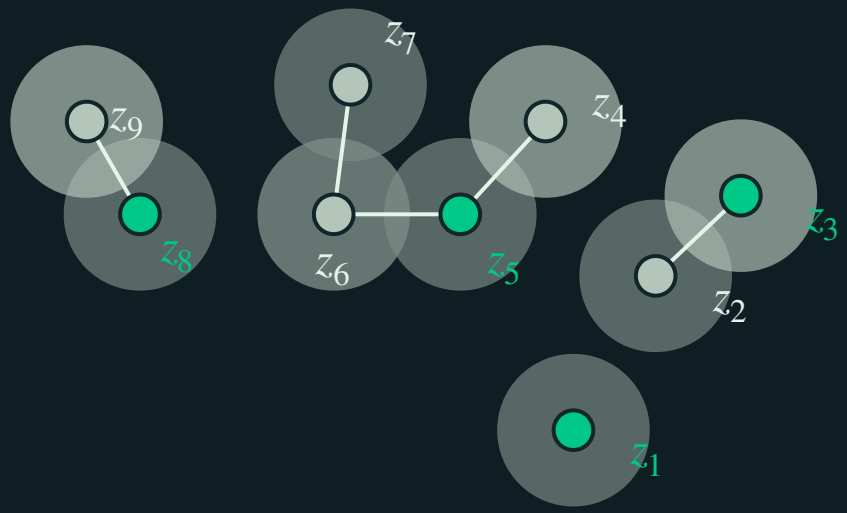
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Size of the candidate set

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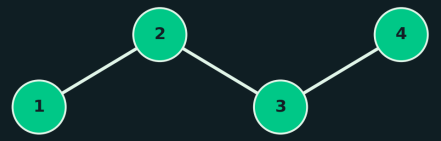
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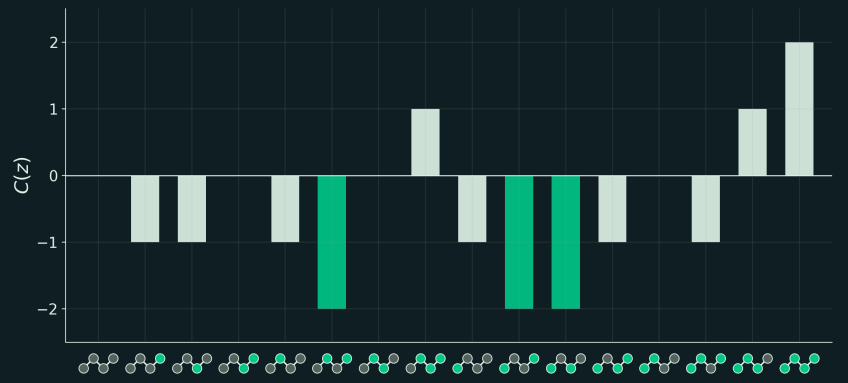
Number of violated edges

$$U \gg 1$$

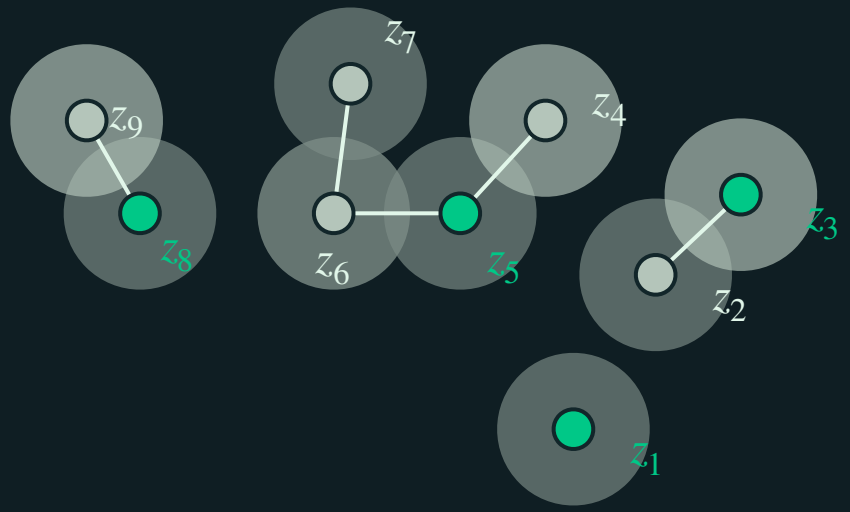
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From a graph to a cost function





configuration z



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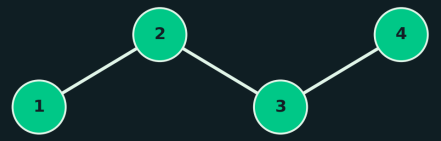
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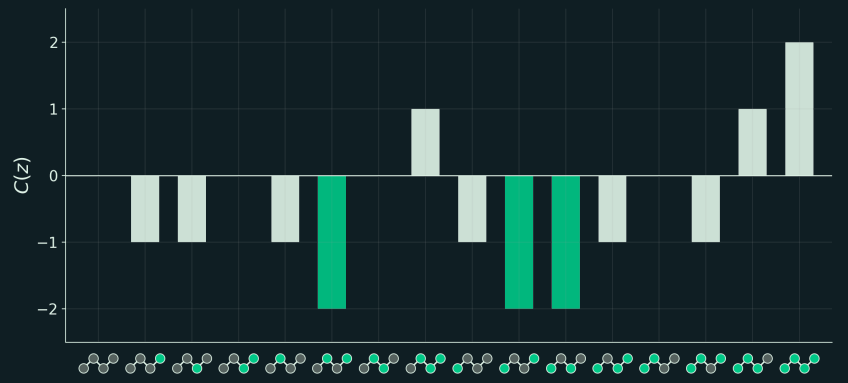
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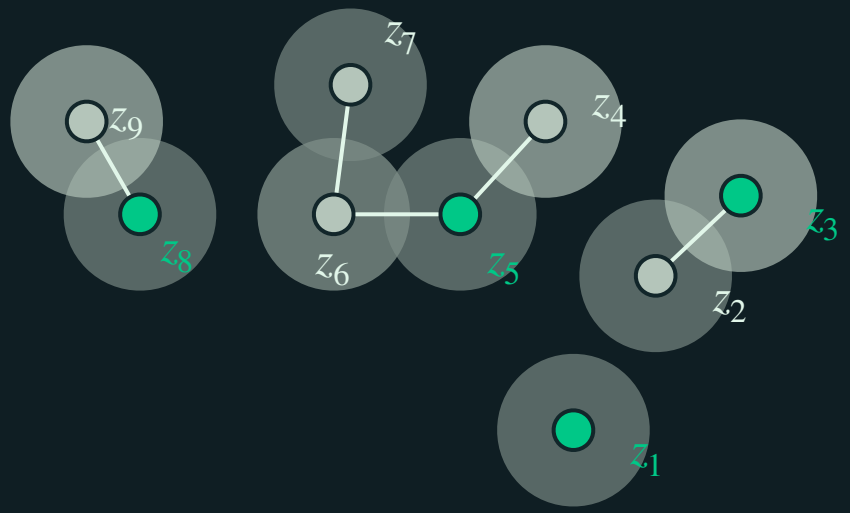
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Theorem: Minimum of C corresponds to a maximum independent set!

From a (classical) cost function to a quantum Hamiltonian

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Computer science

From a (classical) cost function to a quantum Hamiltonian

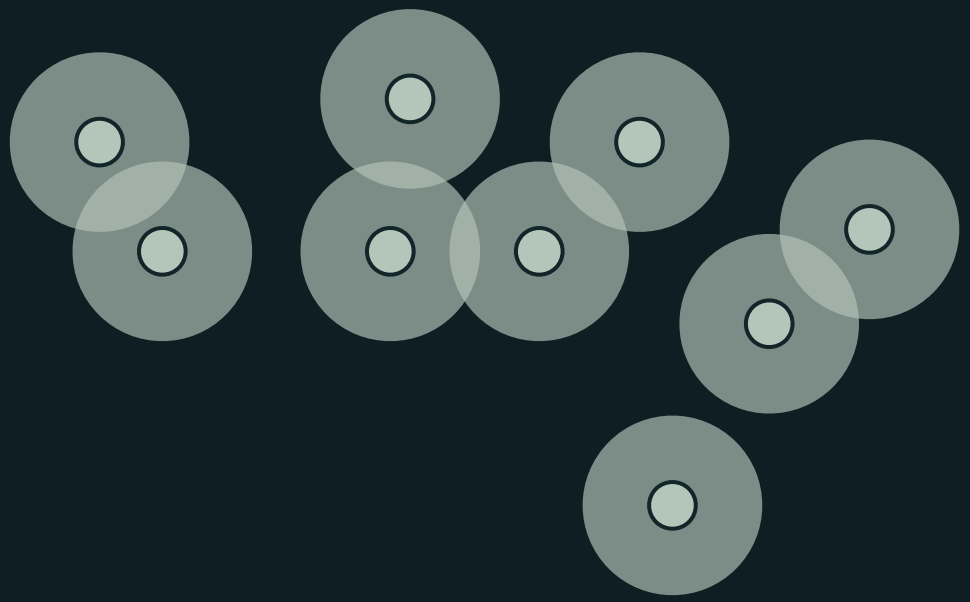
Computer science

Minimum of C, representing the solution of an optimization problem

From a (classical) cost function to a quantum Hamiltonian

Computer science

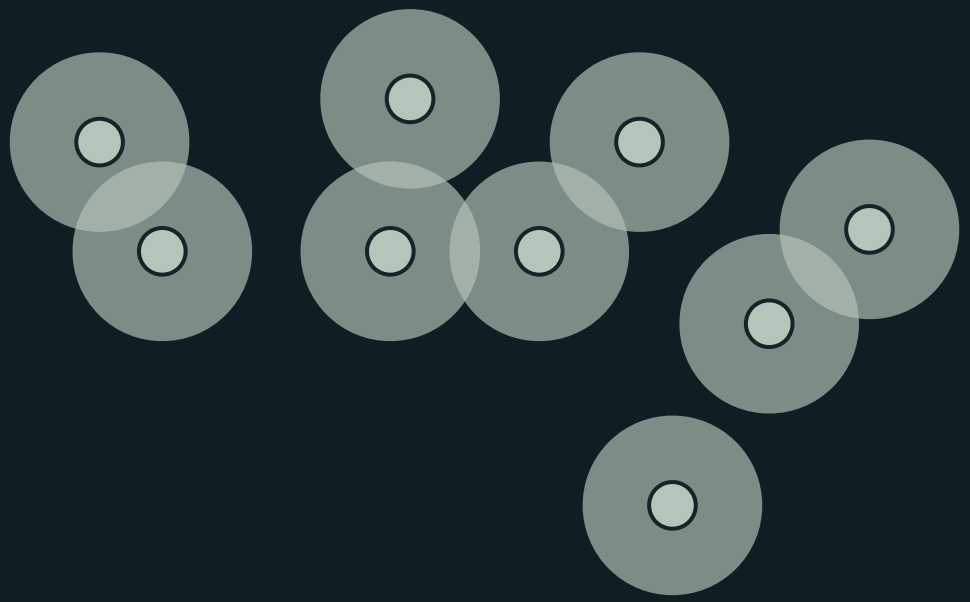
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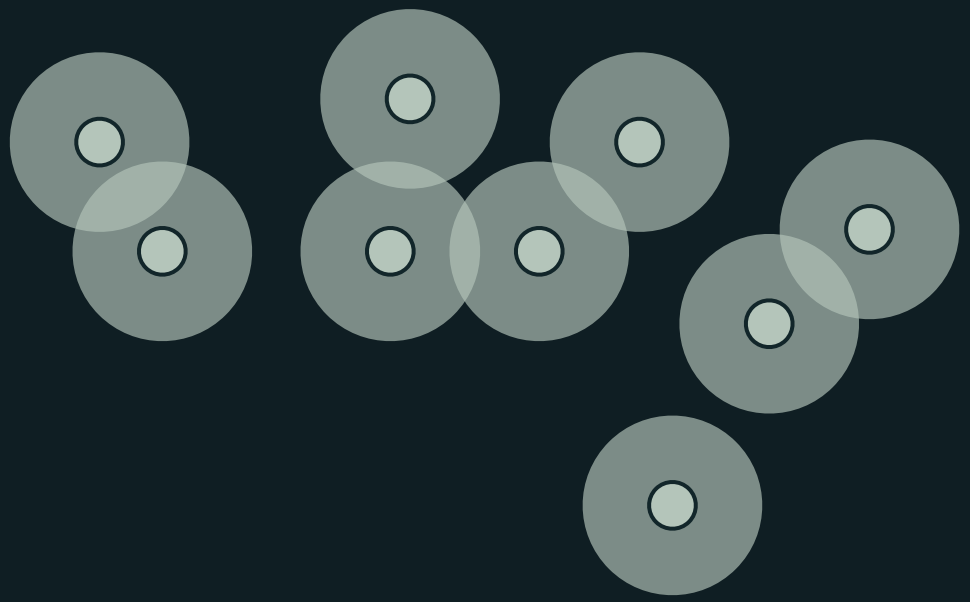
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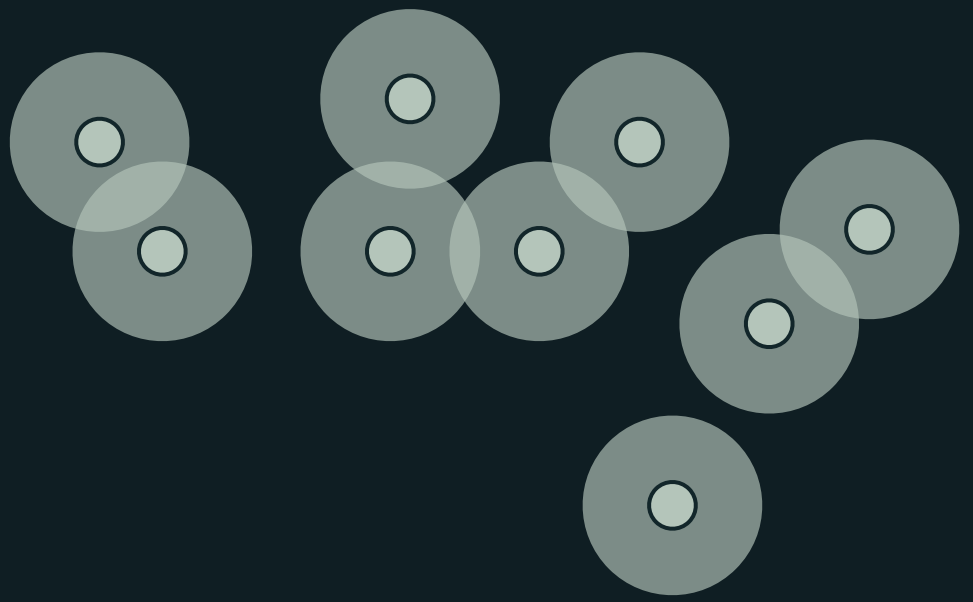


Physics

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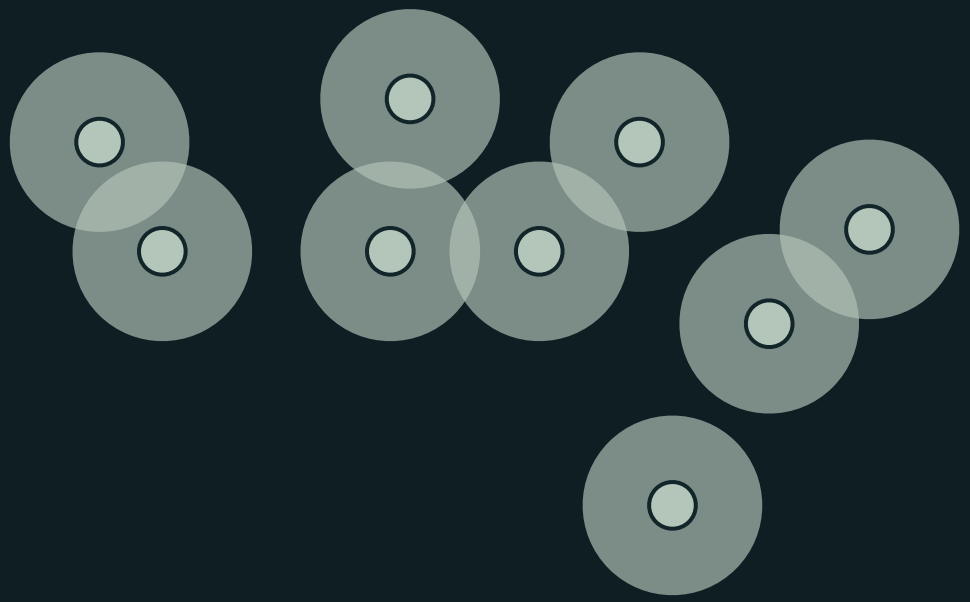
Physics

Ground state of Hamiltonian H_c

From a (classical) cost function to a quantum Hamiltonian

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Ground state of Hamiltonian H_c

Quantum system obeys continuous-time dynamics

From a (classical) cost function to a quantum Hamiltonian

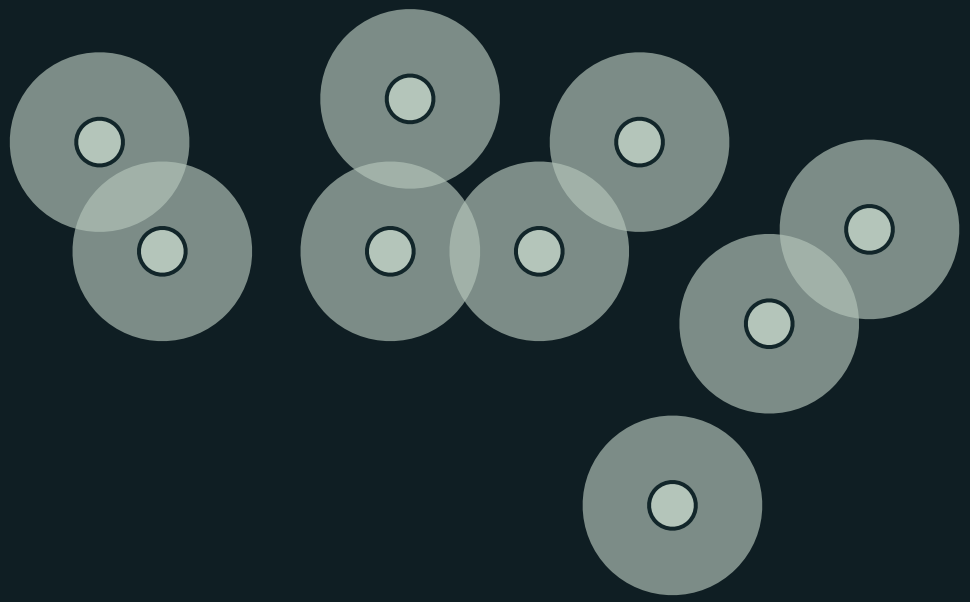
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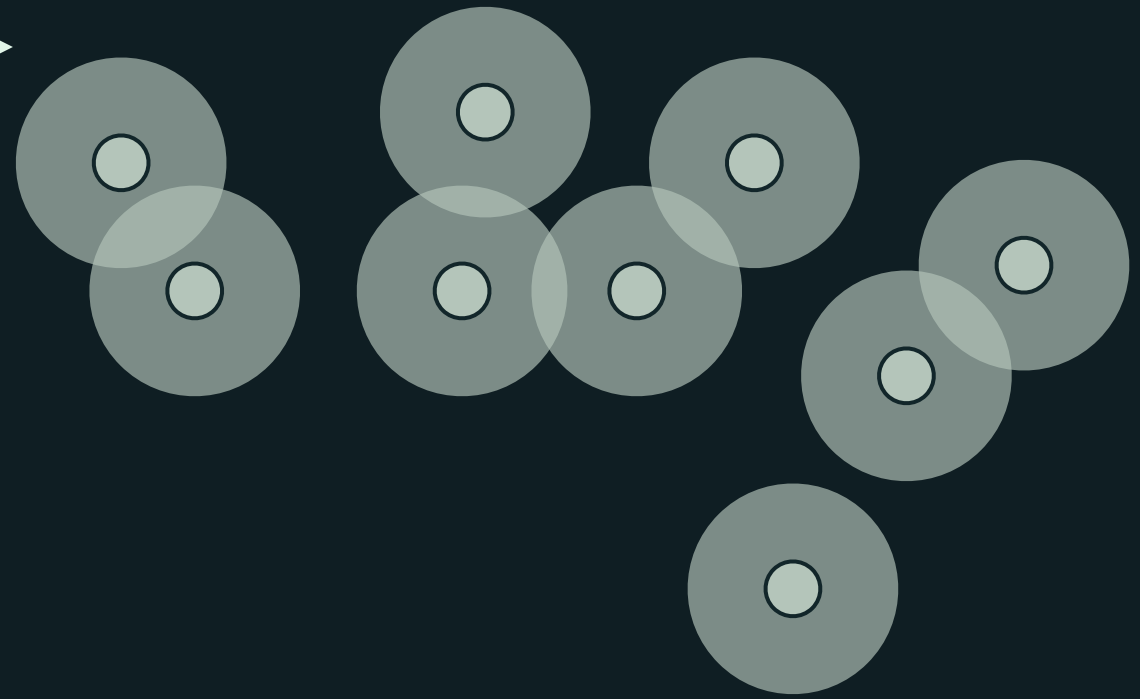
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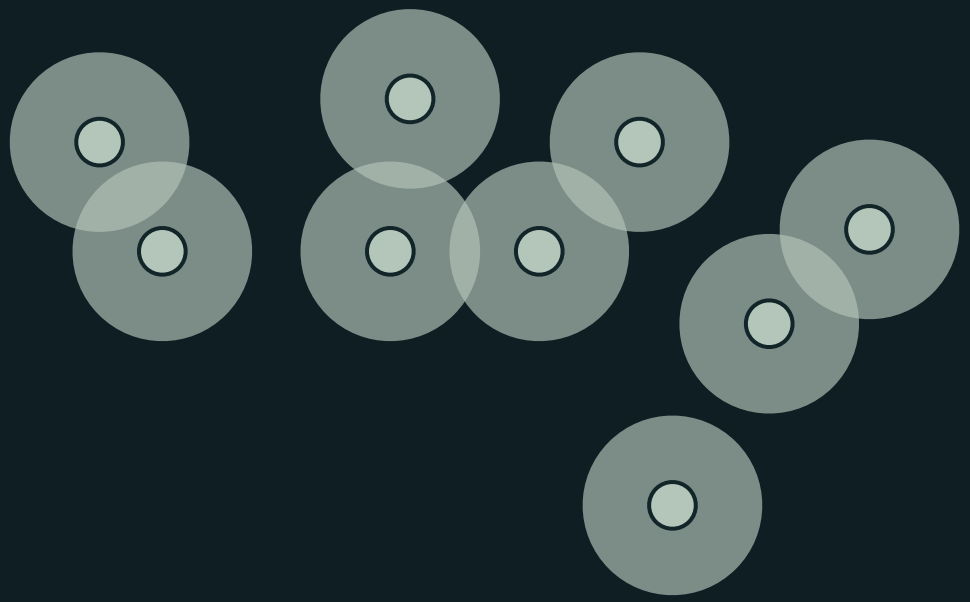




From a (classical) cost function to a quantum Hamiltonian

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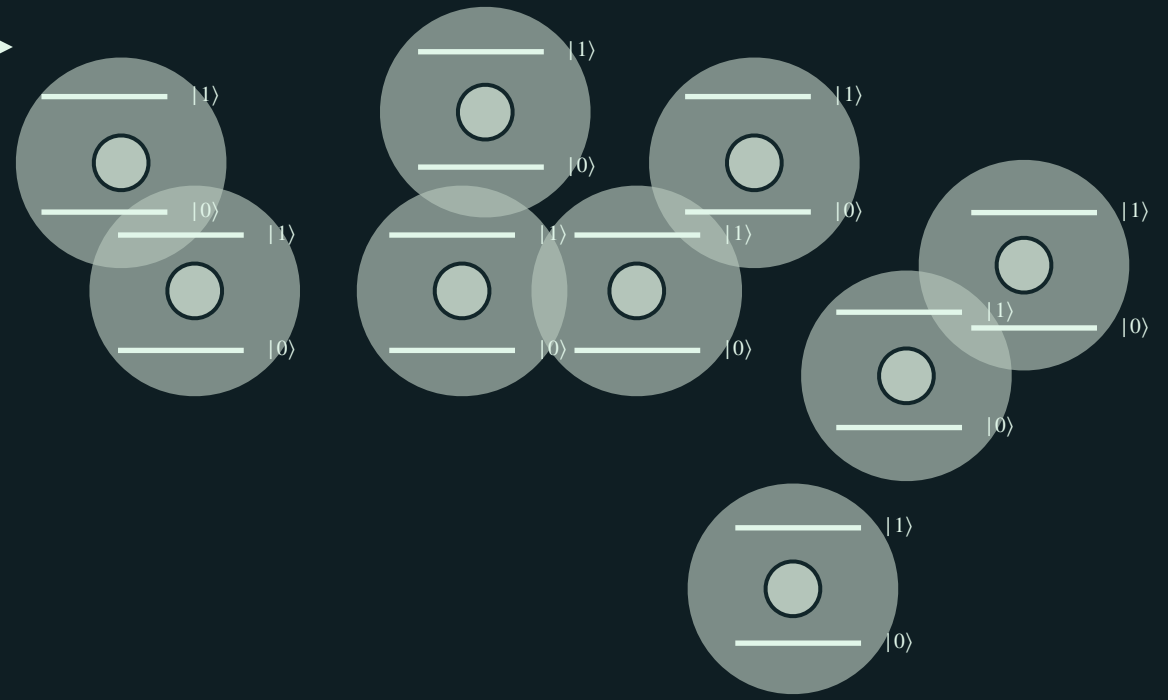
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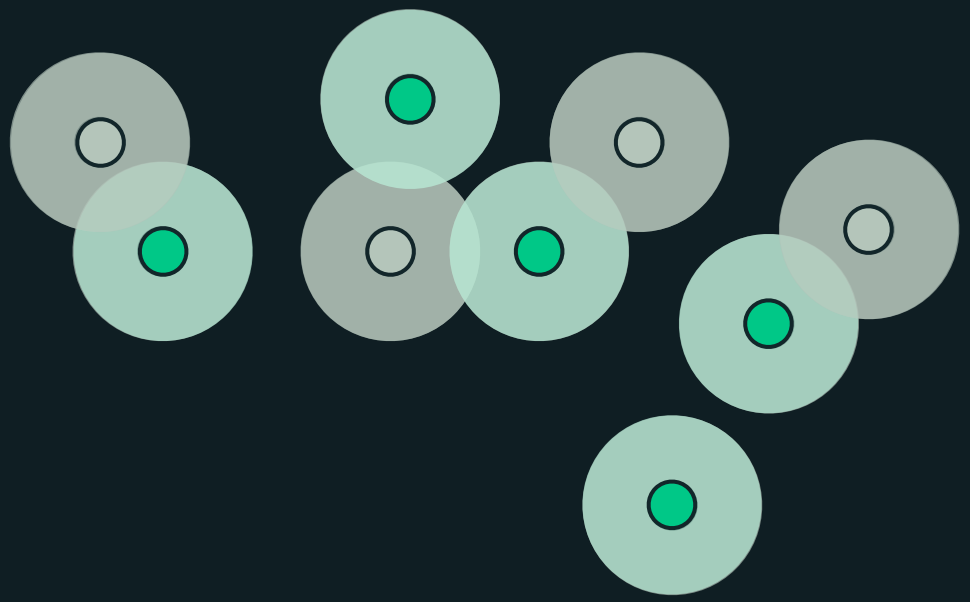
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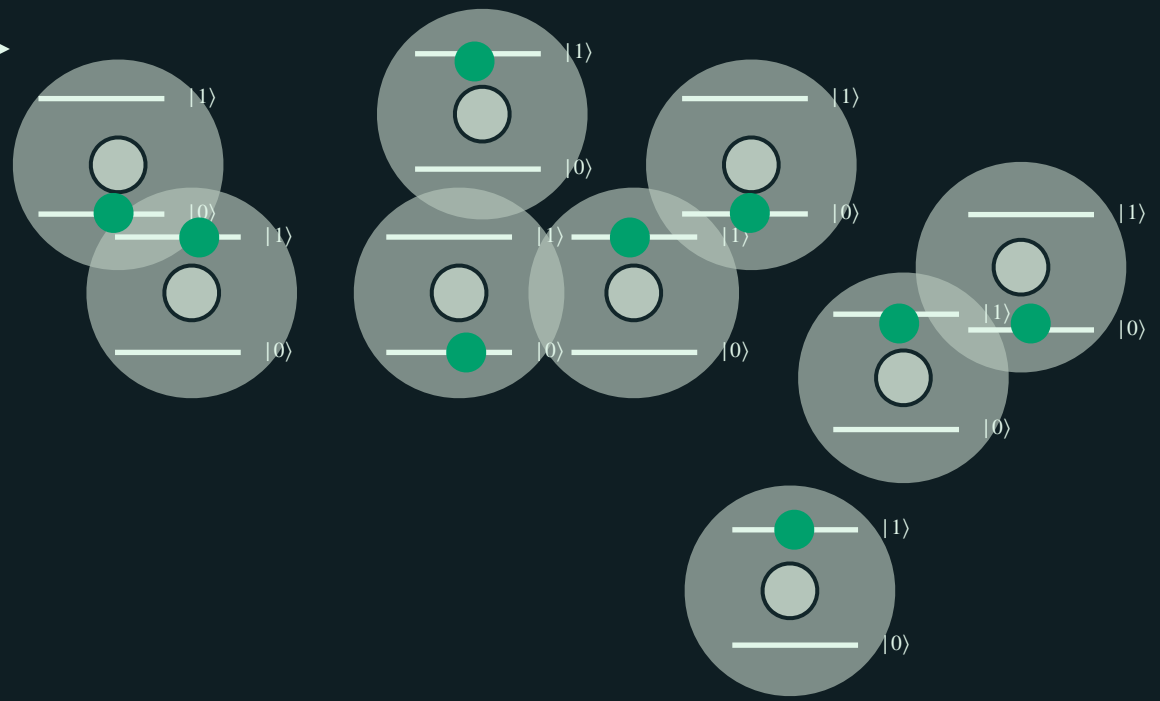
Minimum of C, representing the solution of an optimization problem



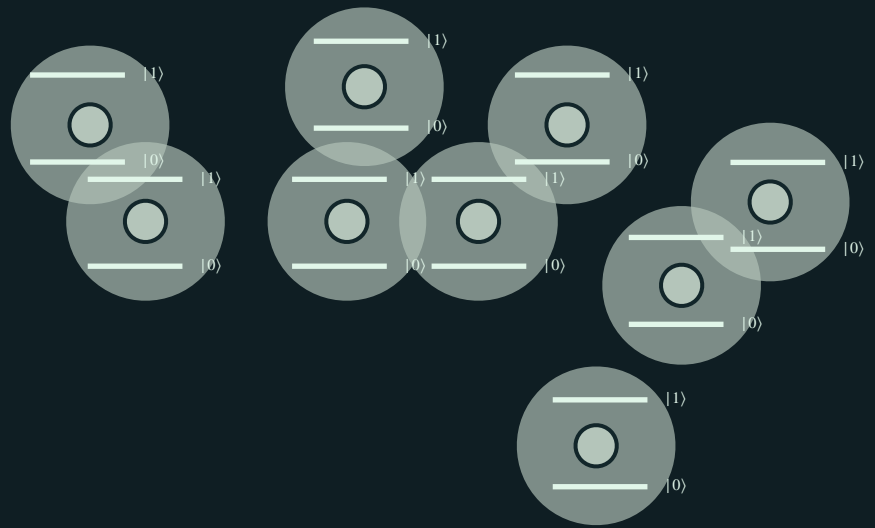
Physics

Ground state of Hamiltonian H_c

Quantum system obeys continuous-time dynamics

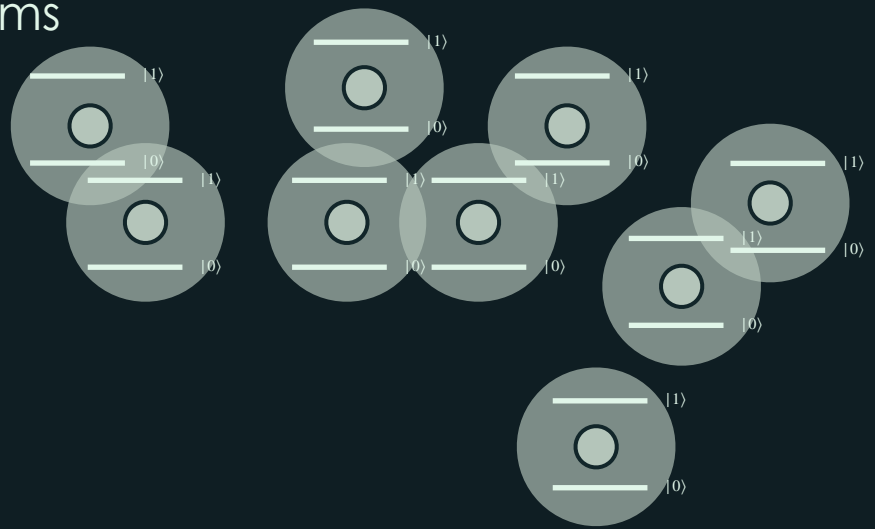


Neutral atom Hamiltonians



Neutral atom Hamiltonians

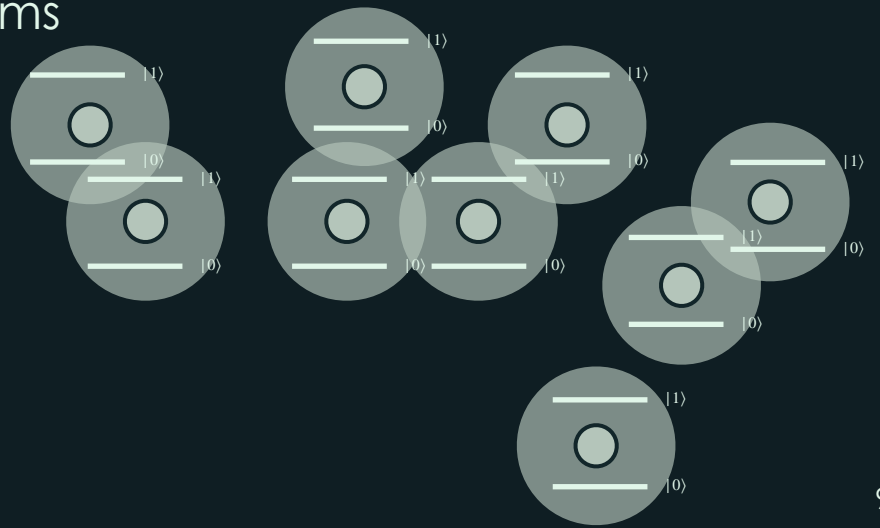
Example Rydberg atoms



Neutral atom Hamiltonians

Example Rydberg atoms



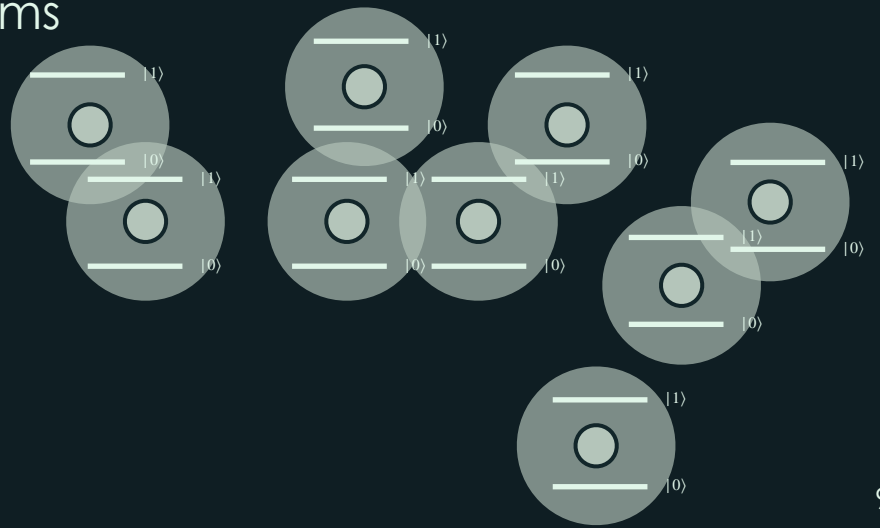


Neutral atom Hamiltonians

$$H =$$

Example Rydberg atoms



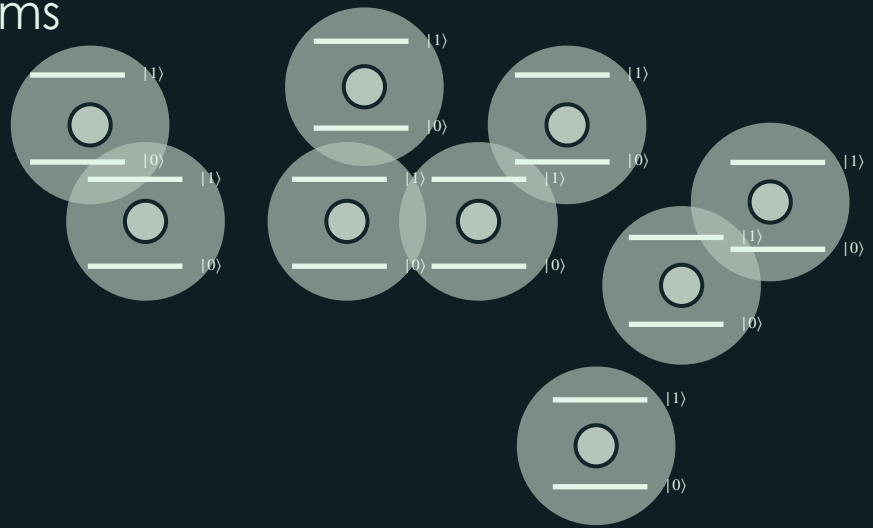


Neutral atom Hamiltonians

$$H = -\delta N$$

Example Rydberg atoms





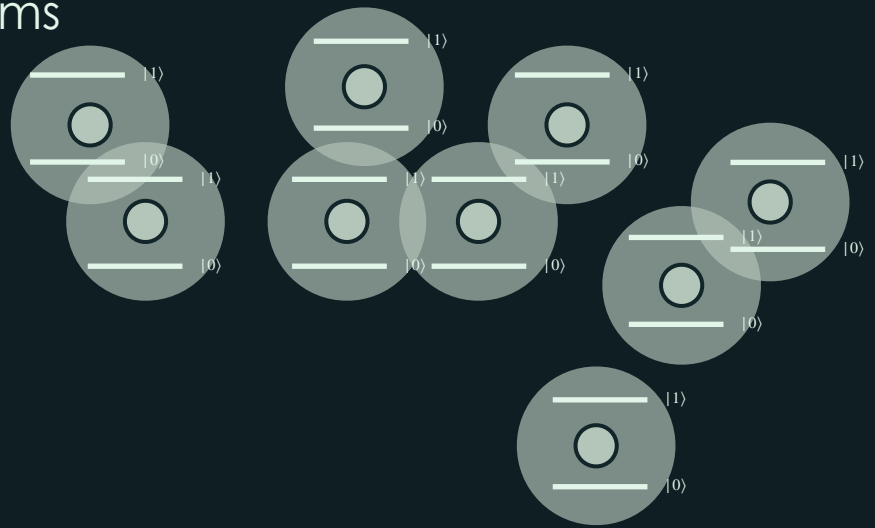
Neutral atom Hamiltonians

$$H = -\delta N$$

Number of excited atoms

Example Rydberg atoms





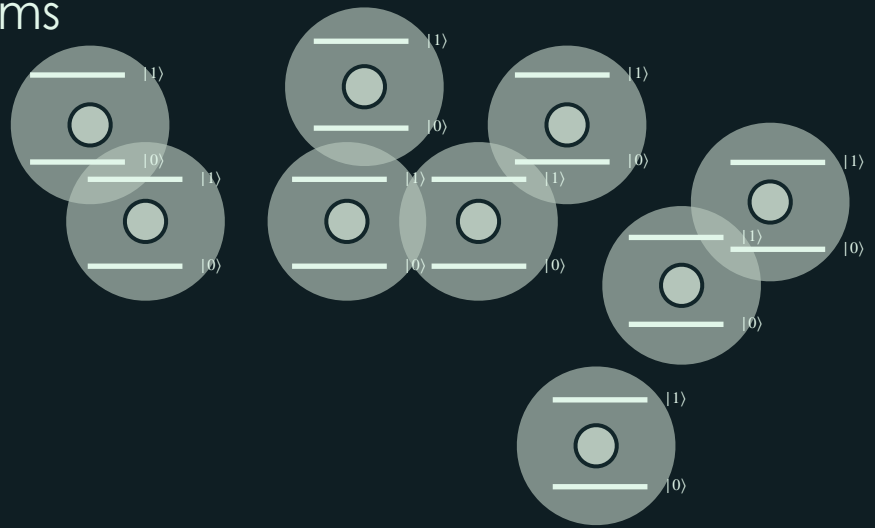
Neutral atom Hamiltonians

$$H = -\delta N +$$

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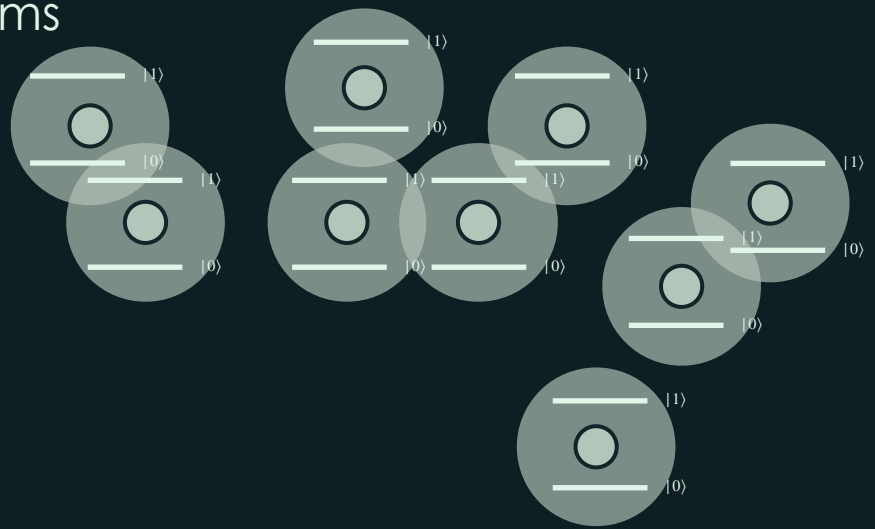
Neutral atom Hamiltonians

$$H = -\delta N + J \times \sum N_1 \otimes N_2$$

Number of excited atoms

Example Rydberg atoms





Neutral atom Hamiltonians

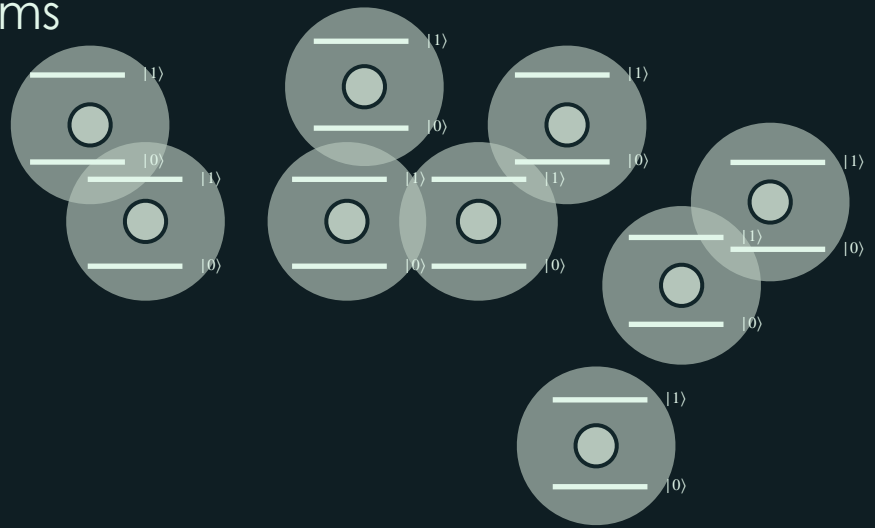
$$H = -\delta N + J \times \sum N_1 \otimes N_2$$

Number of excited atoms

Pair of atoms both excited

Example Rydberg atoms





Neutral atom Hamiltonians

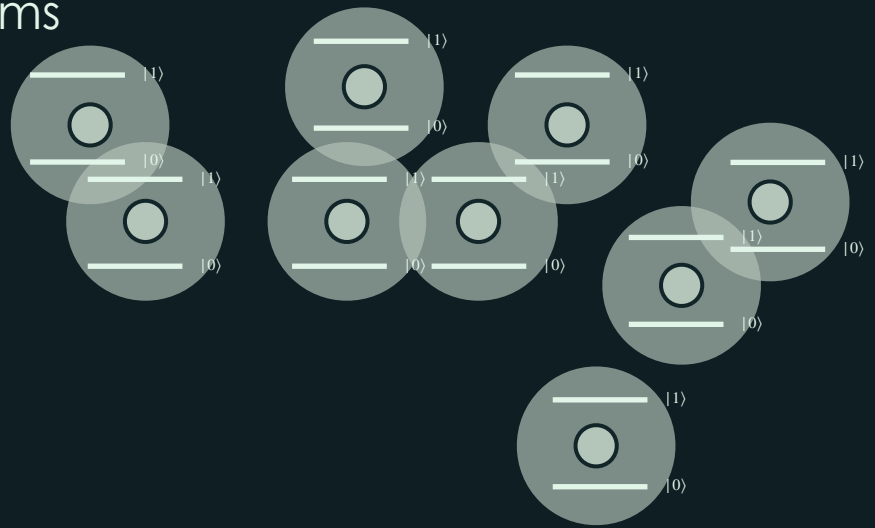
$$H = -\delta N + J \times \sum N_1 \otimes N_2$$

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Neutral atom Hamiltonians

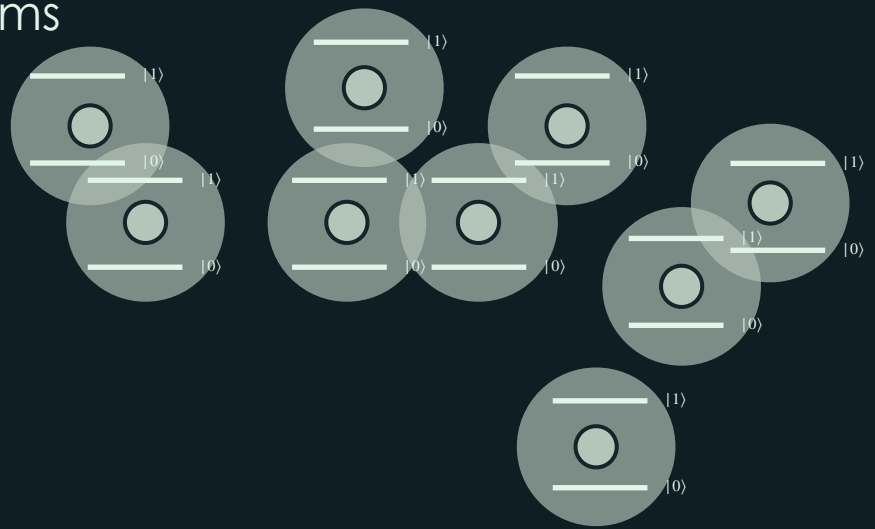
$$H = -\delta N + J \times \sum N_1 \otimes N_2 \approx H_c + \text{unwanted interactions}$$

Number of excited atoms

Pair of atoms both excited

Example Rydberg atoms





Neutral atom Hamiltonians

$$H =$$

$$\approx H_c + \text{unwanted interactions}$$

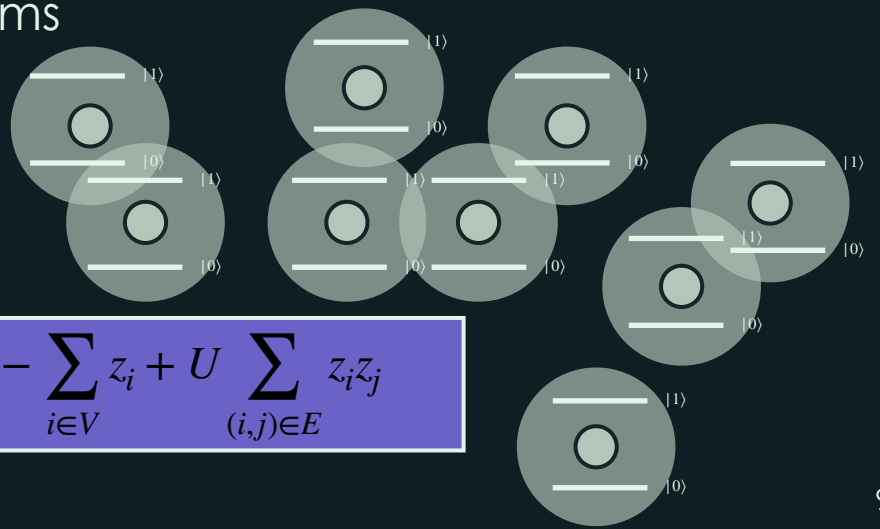
$$-\delta N \quad + \quad J \times \sum N_1 \otimes N_2$$

Number of excited atoms

Pair of atoms both excited

Example Rydberg atoms





$$C(z_1, \ldots, z_n) = -\sum_{i \in V} z_i + U \sum_{(i,j) \in E} z_i z_j$$

Neutral atom Hamiltonians

$$\approx H_c + \text{unwanted interactions}$$

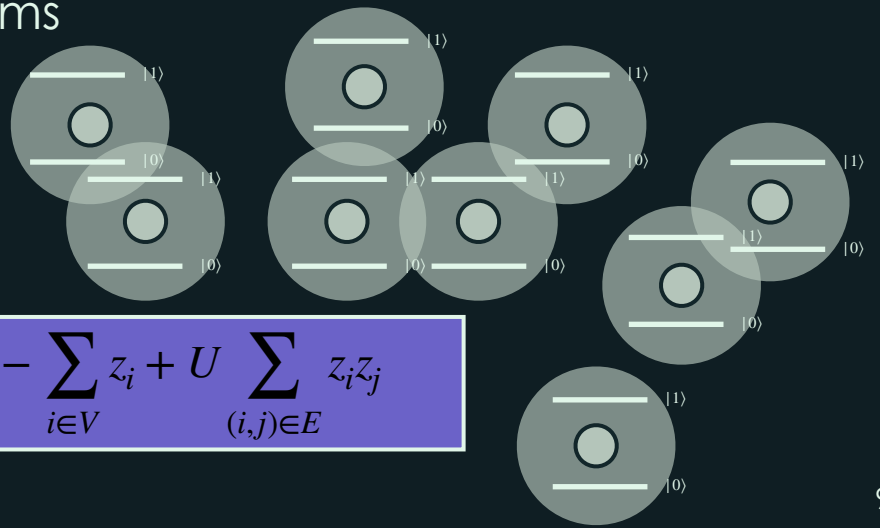
$$H = \quad + \quad -\delta N \quad + \quad J \times \sum N_1 \otimes N_2$$

Number of excited atoms

Pair of atoms both excited

Example Rydberg atoms





$$C(z_1, \ldots, z_n) = -\sum_{i \in V} z_i + U \sum_{(i,j) \in E} z_i z_j$$

Neutral atom Hamiltonians

$$\approx H_c + \text{unwanted interactions}$$

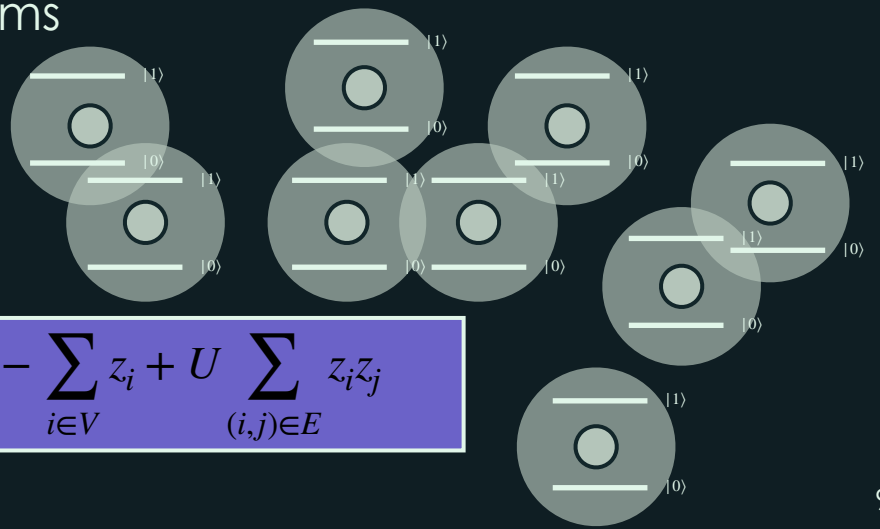
$$H = \Omega X + -\delta N + J \times \sum N_1 \otimes N_2$$

Number of excited atoms

Pair of atoms both excited

Example Rydberg atoms





$$C(z_1, \ldots, z_n) = -\sum_{i \in V} z_i + U \sum_{(i,j) \in E} z_i z_j$$

Neutral atom Hamiltonians

$$\approx H_c + \text{unwanted interactions}$$

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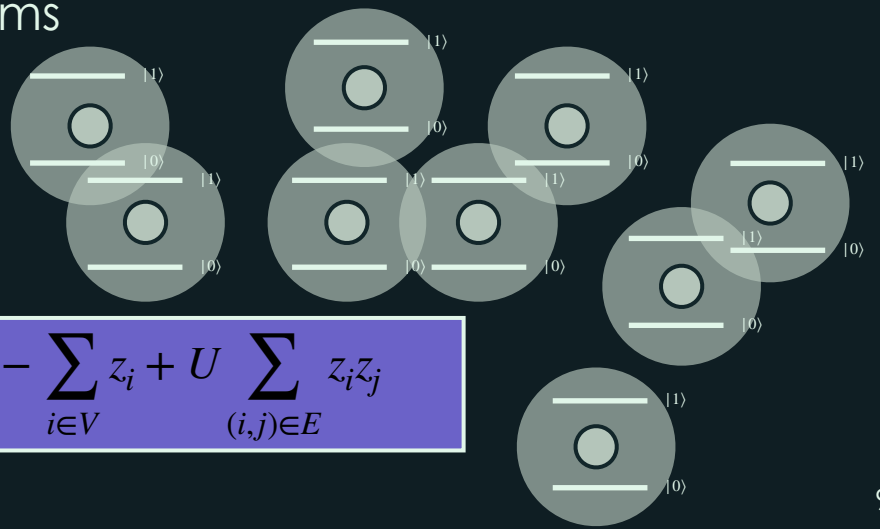
Flip the atoms
Mixing, exploration phase
Control their state with laser

Number of excited atoms

Pair of atoms both excited

Example Rydberg atoms





$$C(z_1, \ldots, z_n) = -\sum_{i \in V} z_i + U \sum_{(i,j) \in E} z_i z_j$$

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$$\approx H_c + \text{unwanted interactions}$$

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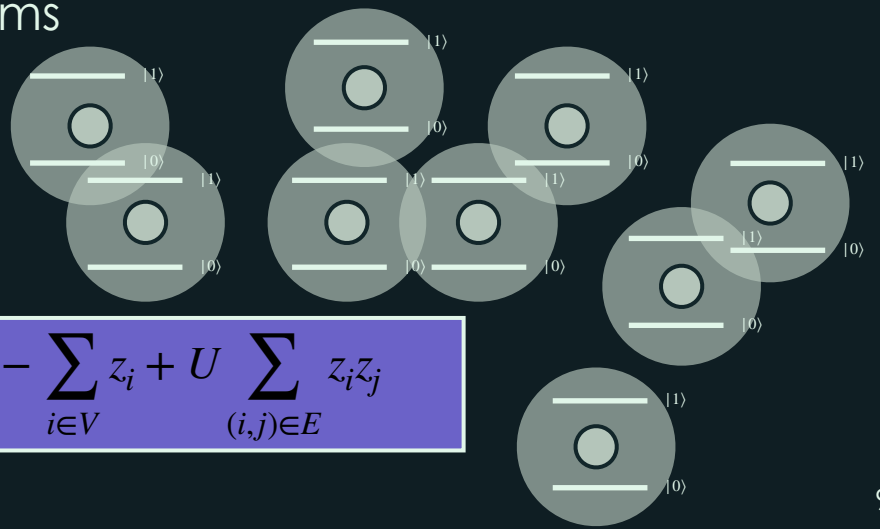
Flip the atoms
Mixing, exploration phase
Control their state with laser

Number of excited atoms

Pair of atoms both excited

Example Rydberg atoms





$$H = \frac{1}{2}\sum_{i=1}^{n} \Omega_i X_i - \sum_{i=1}^{n} \delta_i N_i + \sum_{i<j} \frac{C_6}{d_{ij}^6} N_i \otimes N_j$$

$$C(z_1, \ldots, z_n) = -\sum_{i \in V} z_i + U \sum_{(i,j) \in E} z_i z_j$$

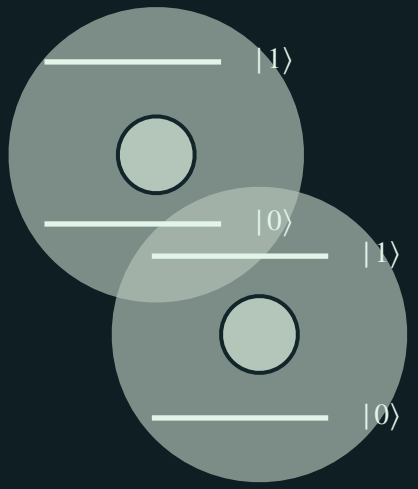
Pasqal Defining the quantum reality

How to run our algorithm on the quantum computer?

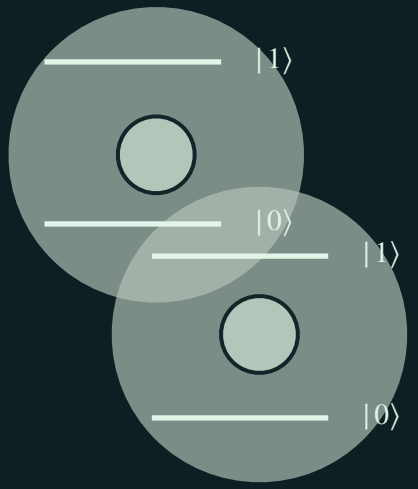
[BVC+25] Elie Bermot, Lucia Valor, Wesley Coelho, Louis-Paul Henry, and Natalie Pearson. Local Rydberg blockade regimes for disk graph embedding and quantum optimization, 2025

How do 2 quantum dancers interact?

How do 2 quantum dancers interact?

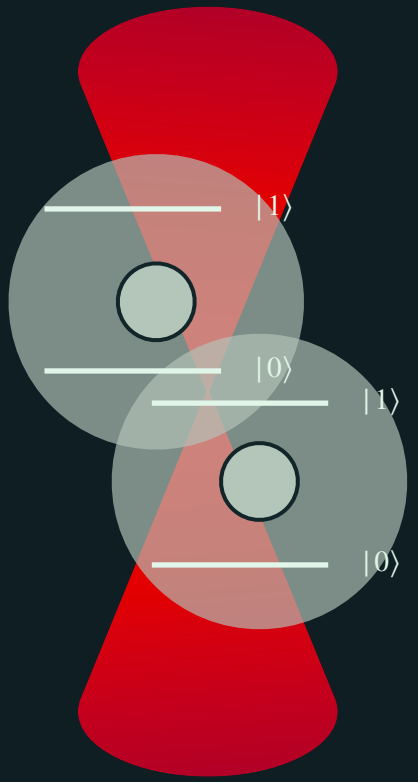


How do 2 quantum dancers interact?



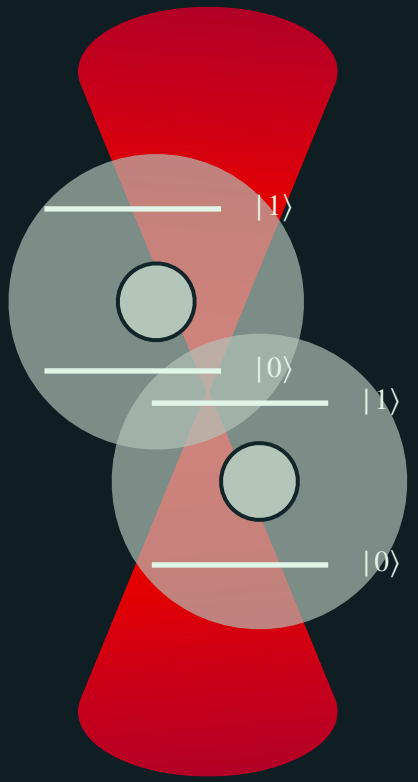
$H = \Omega \cdot$ (flip the atoms) + $1/r^6 N_1 \otimes N_2$

How do 2 quantum dancers interact?

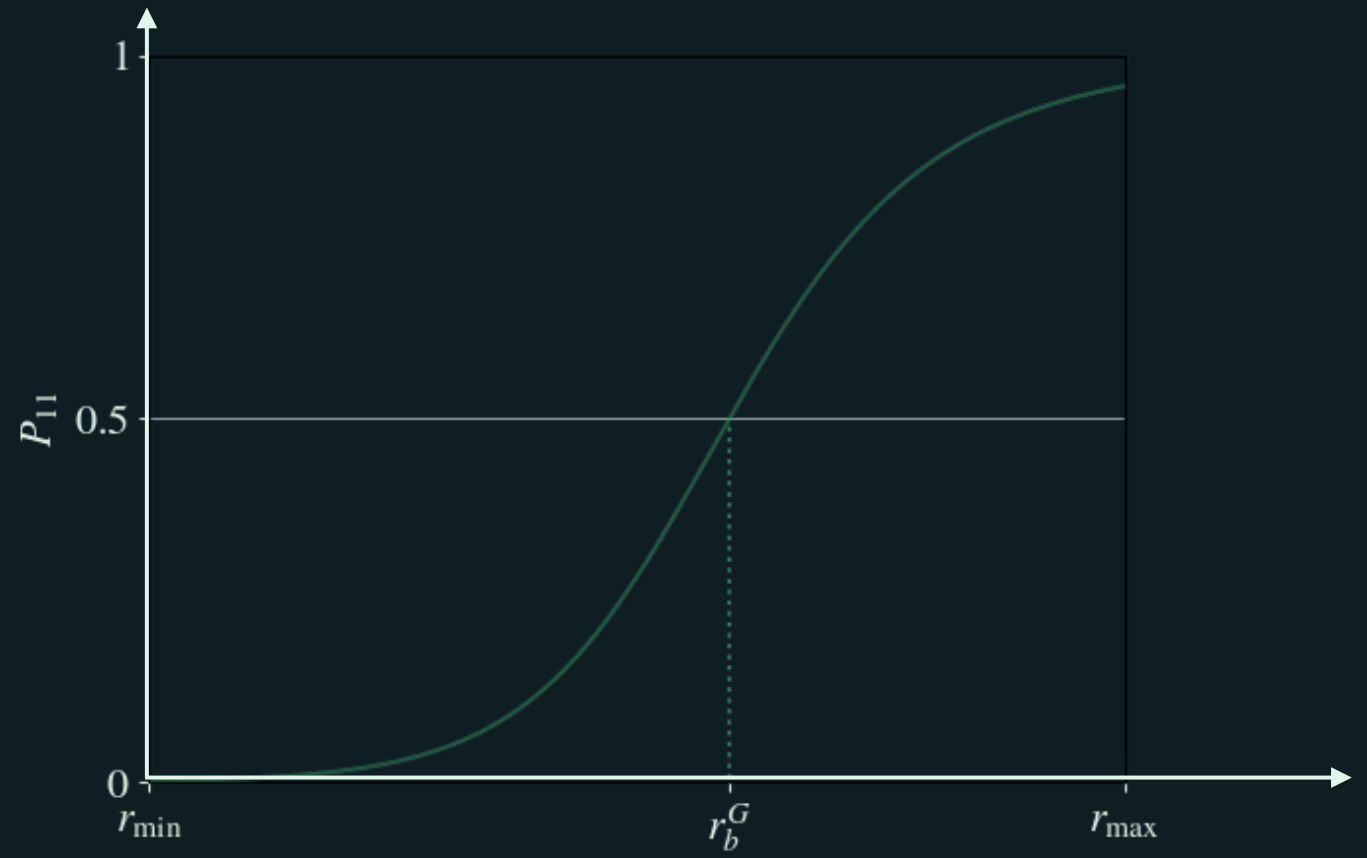


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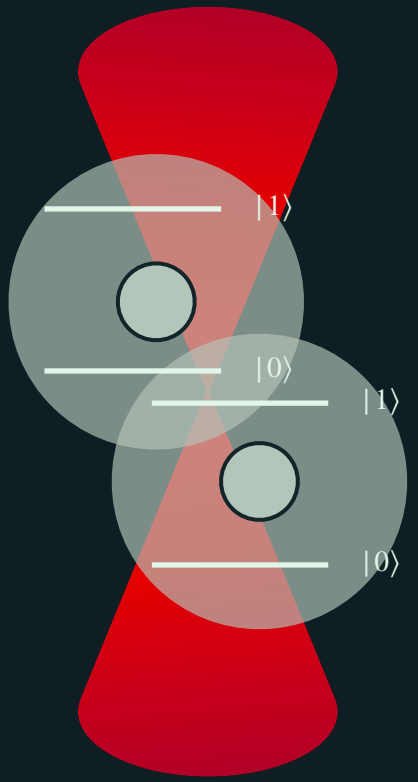


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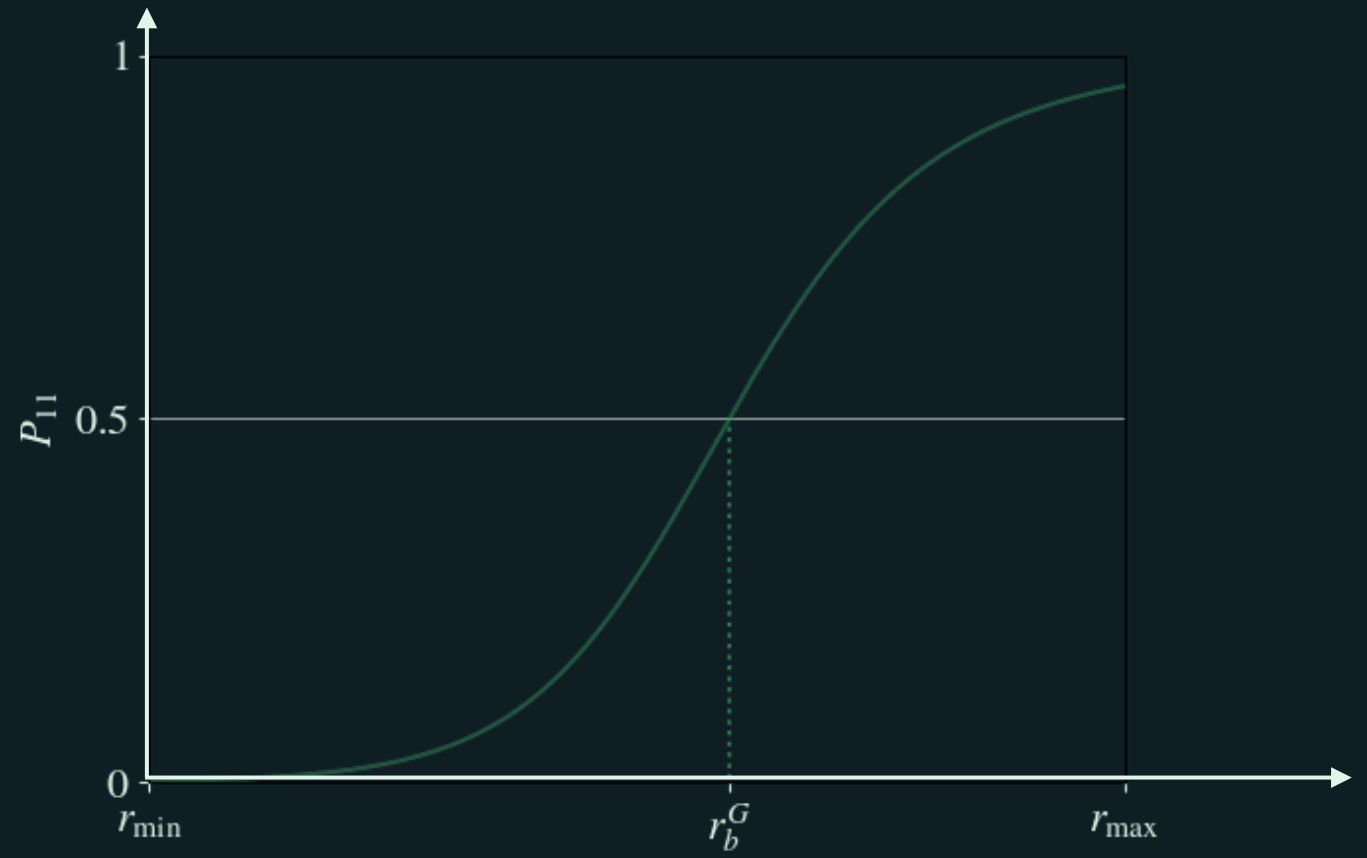


Atoms close r Atoms far apart

How do 2 quantum dancers interact?

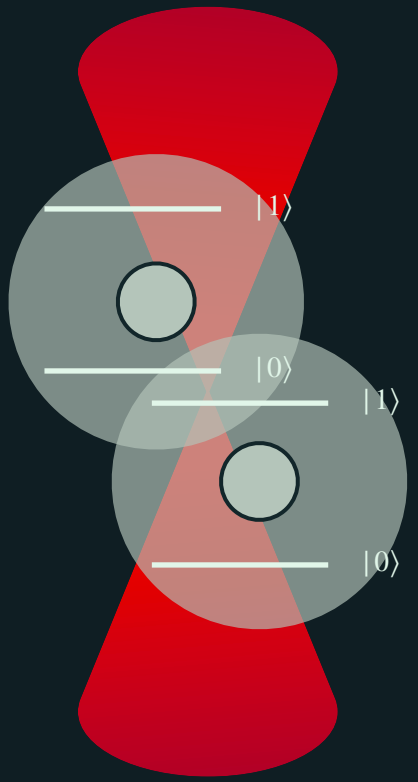


$H = \Omega \cdot (\text{flip the atoms}) + 1/r^6 N_1 \otimes N_2$

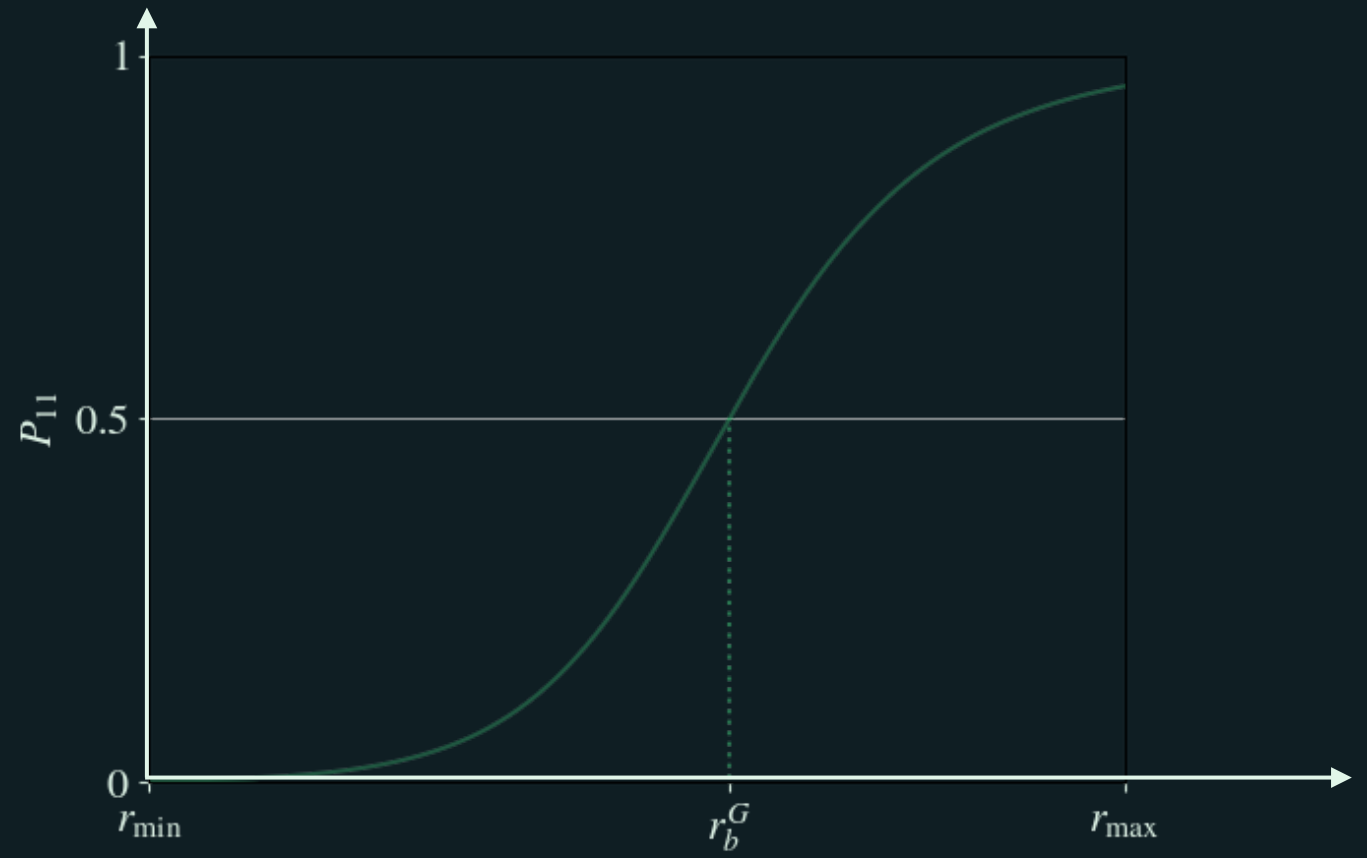


Atoms close r Atoms far apart

How do 2 quantum dancers interact?



$H = \Omega \cdot (\text{flip the atoms}) + 1/r^6 N_1 \otimes N_2$



Atoms close r Atoms far apart

Going from dancers to unit-disk graphs

Going from dancers to unit-disk graphs



Dancers

Going from dancers to unit-disk graphs



Going from dancers to unit-disk graphs



Dancers

Embedding

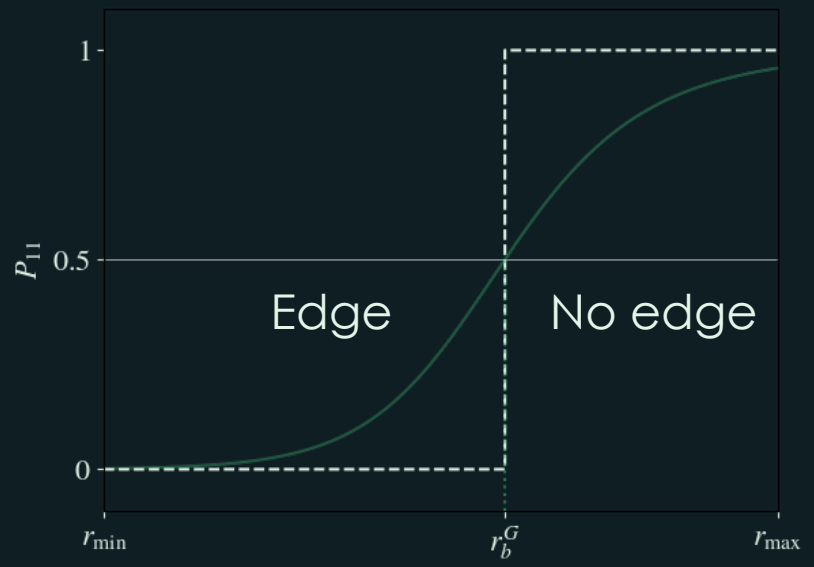


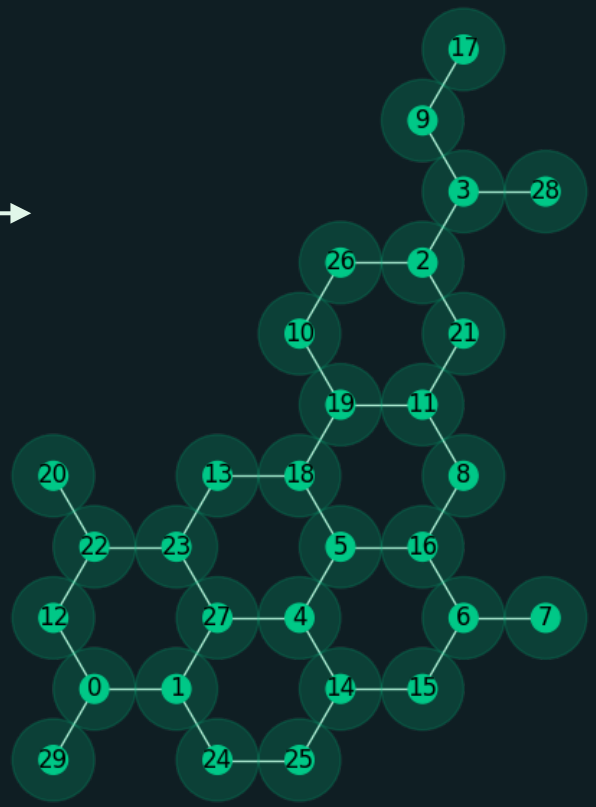
Going from dancers to unit-disk graphs



Dancers

Embedding





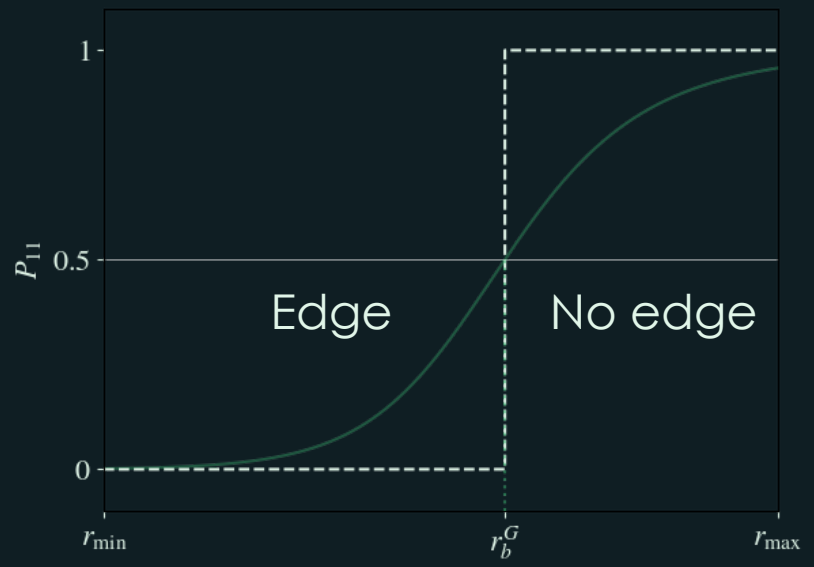
Unit-Disk graph

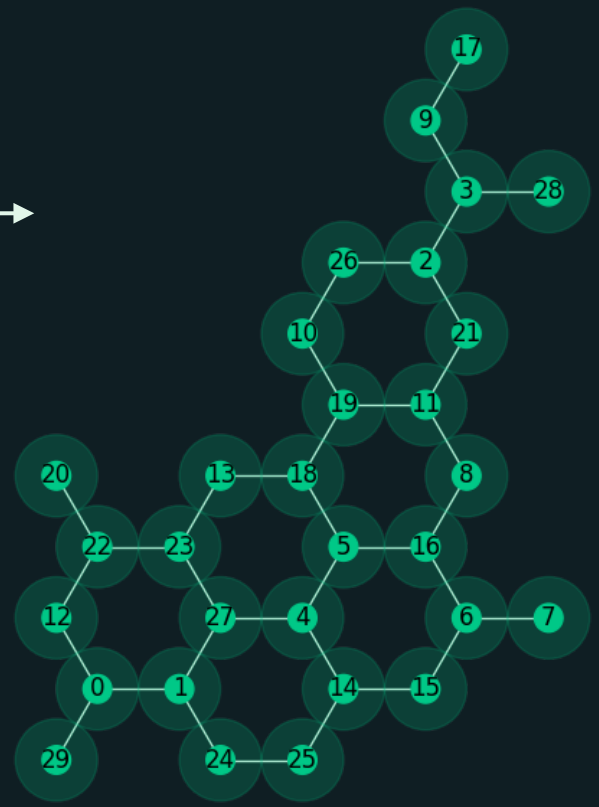
Going from dancers to unit-disk graphs



Dancers

Embedding





Unit-Disk graph

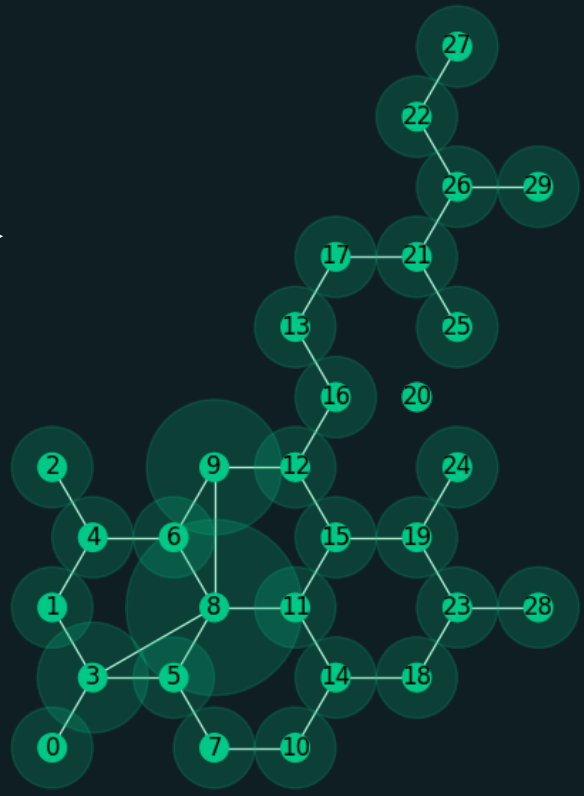
Works for dancers that have same arm length
Can we do better?

Going from dancers to disk graphs



Dancers

Embedding



Disk graphs

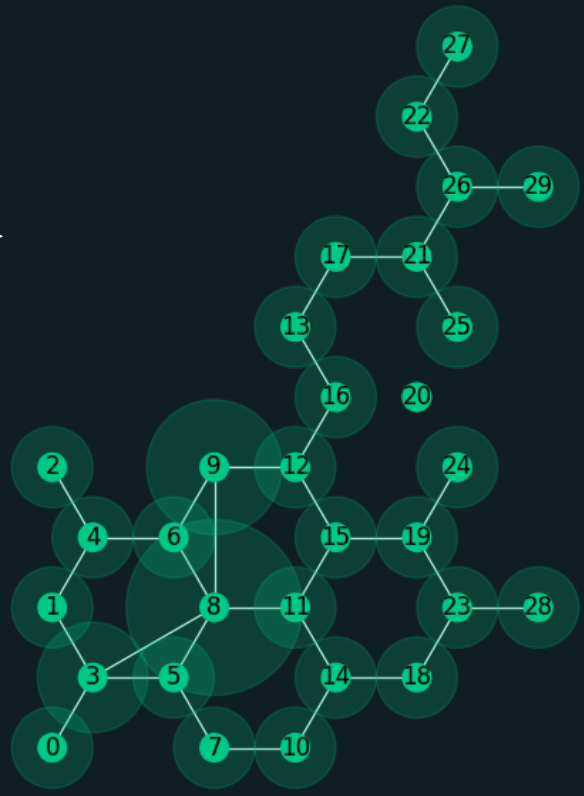
Going from dancers to disk graphs



Dancers

Embedding

Not known!



Disk graphs

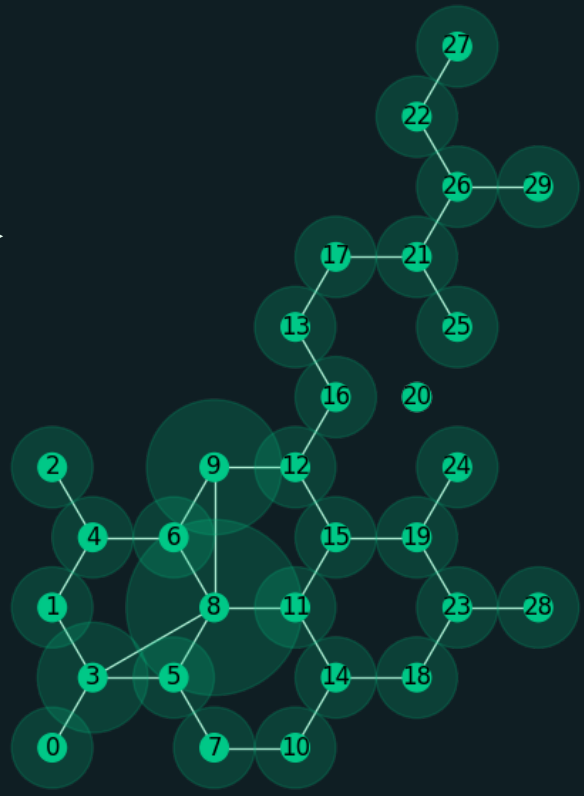
Going from dancers to disk graphs



Dancers

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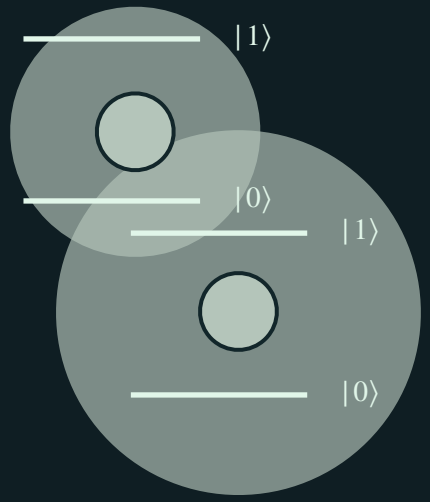


Disk graphs

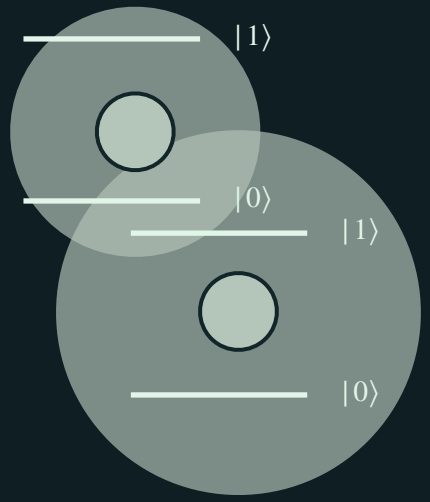
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How do 2 quantum dancers interact? - Our contribution

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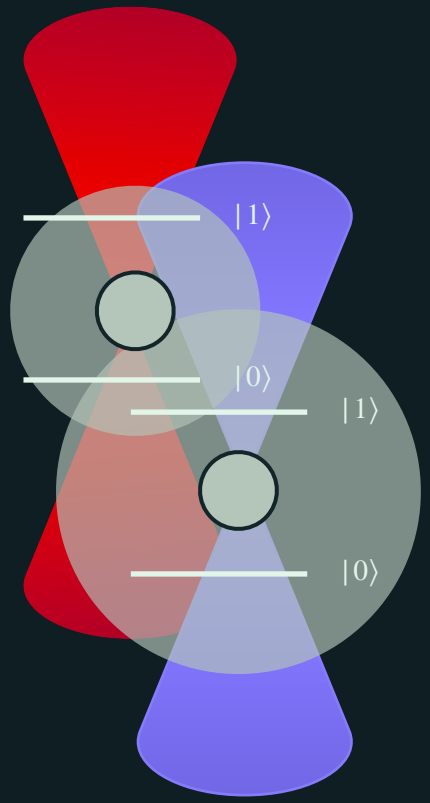


How do 2 quantum dancers interact? - Our contribution



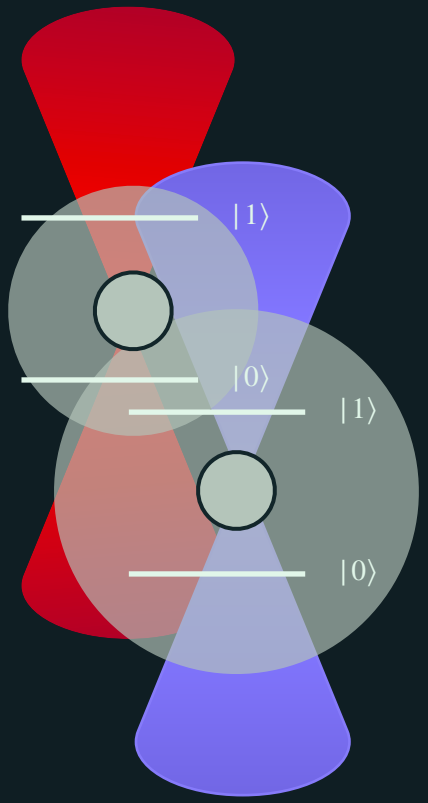
$$H = \Omega_1 \cdot (\text{flip atom 1}) + \Omega_2 \cdot (\text{flip atom 2}) + 1/r^6 N_1 \otimes N_2$$

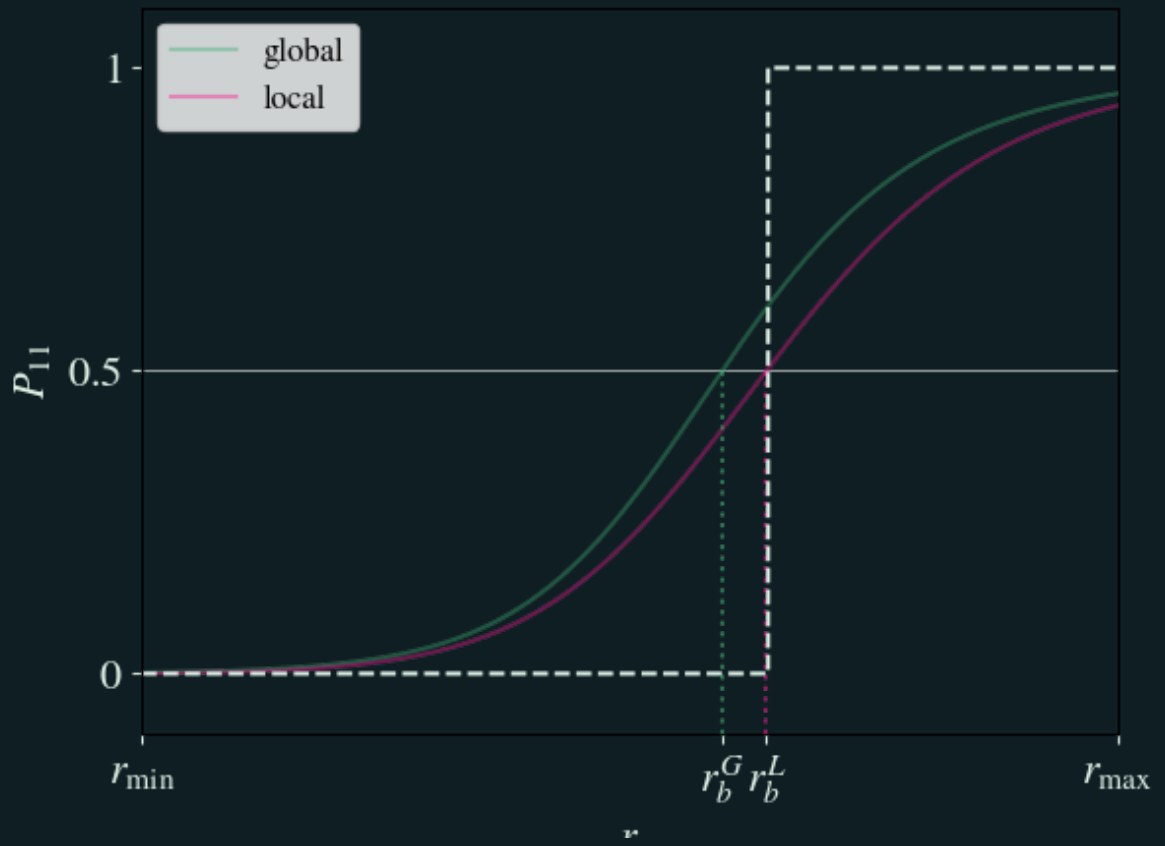
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$$H = \Omega_1 \cdot (\text{flip atom 1}) + \Omega_2 \cdot (\text{flip atom 2}) + 1/r^6 N_1 \otimes N_2$$

Embedding characterization - Our contribution

Embedding characterization - Our contribution

Good embedding: $|\psi\rangle$ only lives on independent set subspaces of G

Embedding characterization - Our contribution

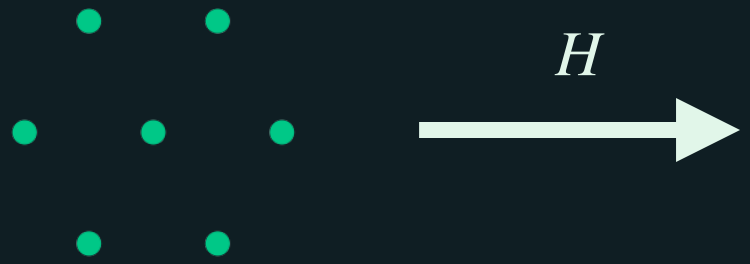
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Register of dancers

Embedding characterization - Our contribution

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Register of dancers

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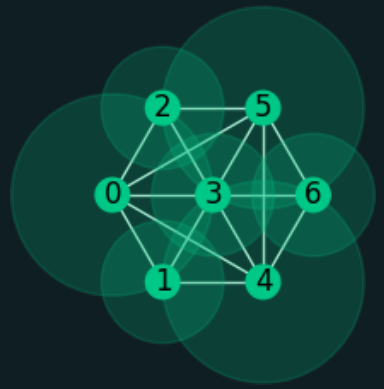
Register of dancers

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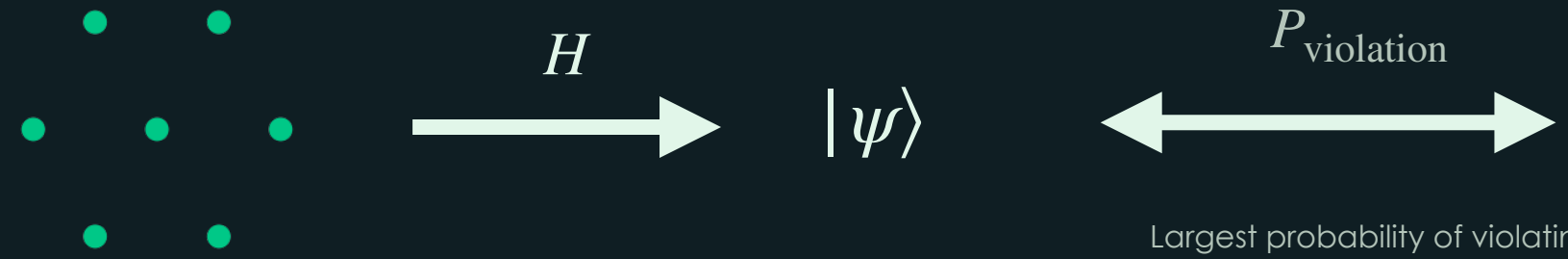
Register of dancers



Target graph of dancers

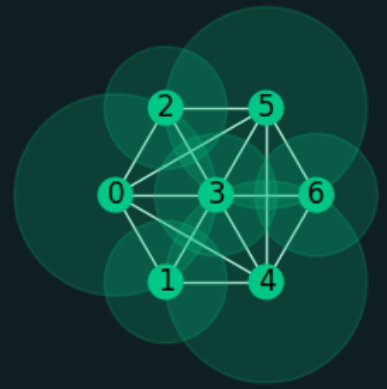
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Register of dancers

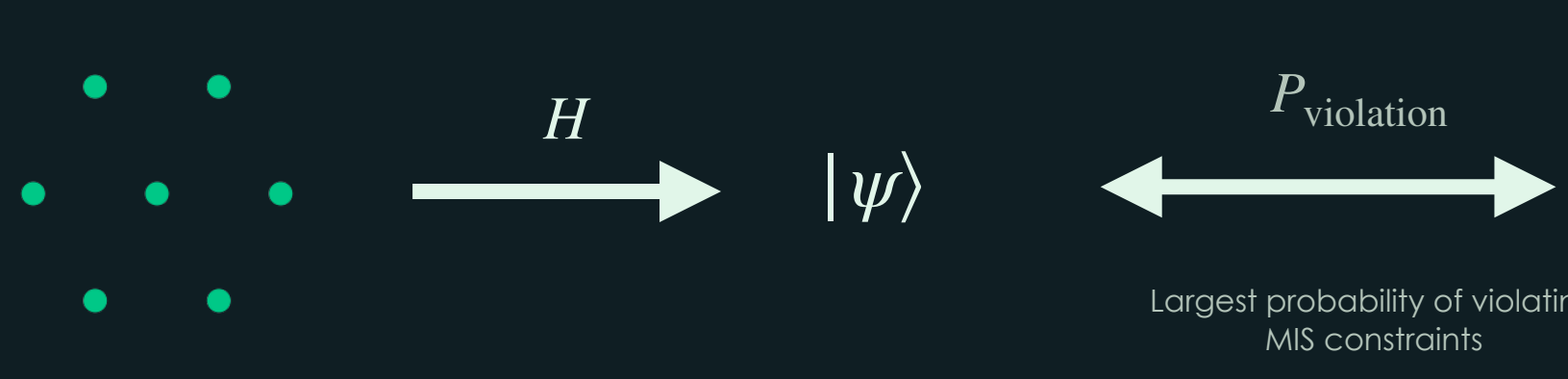
Largest probability of violating MIS constraints



Target graph of dancers

Embedding characterization - Our contribution

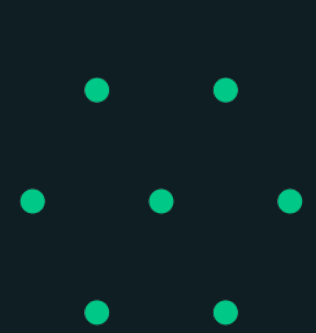
Good embedding: $|\psi\rangle$ **only lives on independent set subspaces of** G



Robustness: uniform dilation of register: all positions $\times \lambda$

Embedding characterization - Our contribution

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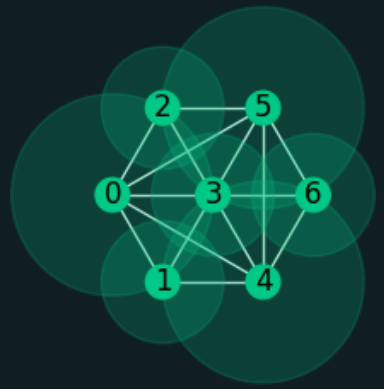
Register of dancers

H

$|\psi\rangle$

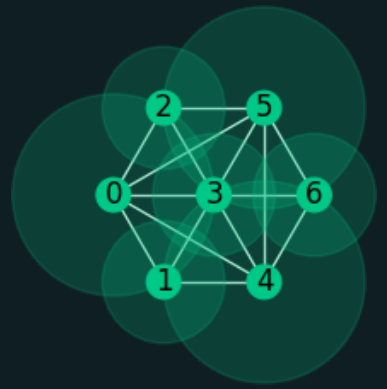
$P_{\text{violation}}$

Largest probability of violating MIS constraints



Target graph of dancers

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Embedding characterization - Our contribution

Good embedding: $|\psi\rangle$ **only lives on independent set subspaces of** G



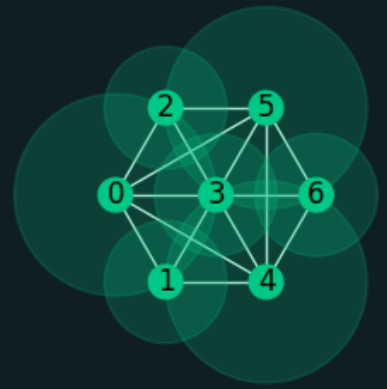
Register of dancers

H

$|\psi\rangle$

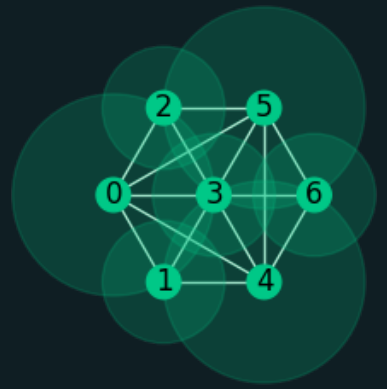
$P_{\text{violation}}$

Largest probability of violating MIS constraints



Target graph of dancers

Robustness: uniform dilation of register: all positions $\times \lambda$



$\lambda < \lambda_c$

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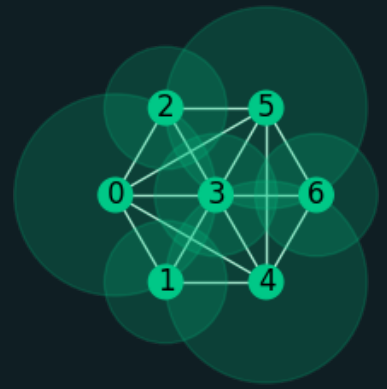
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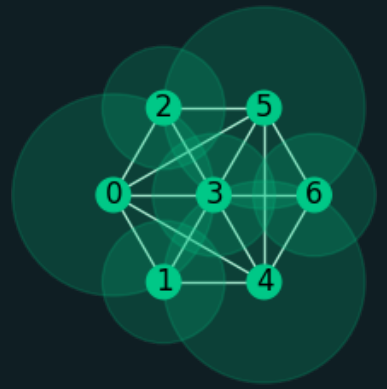
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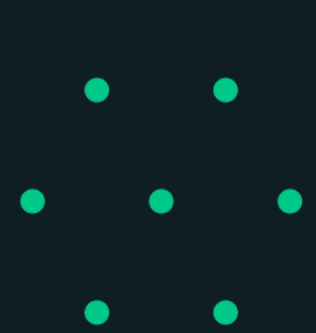


$\lambda < \lambda_c$

λ

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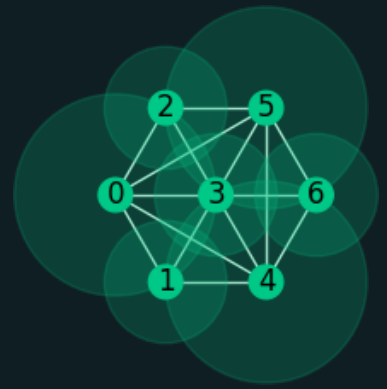
Register of dancers

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$|\psi\rangle$

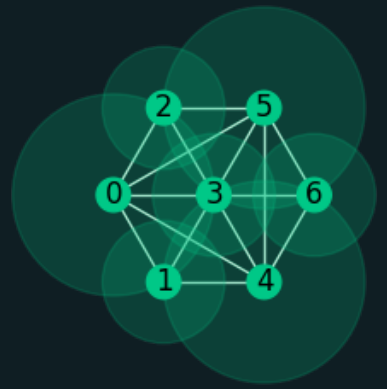
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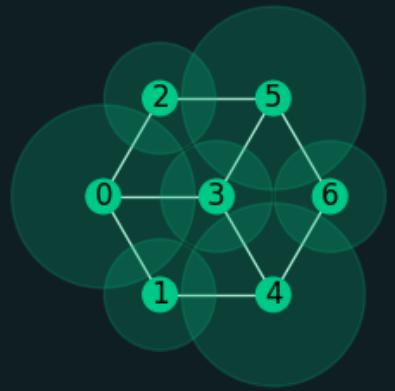
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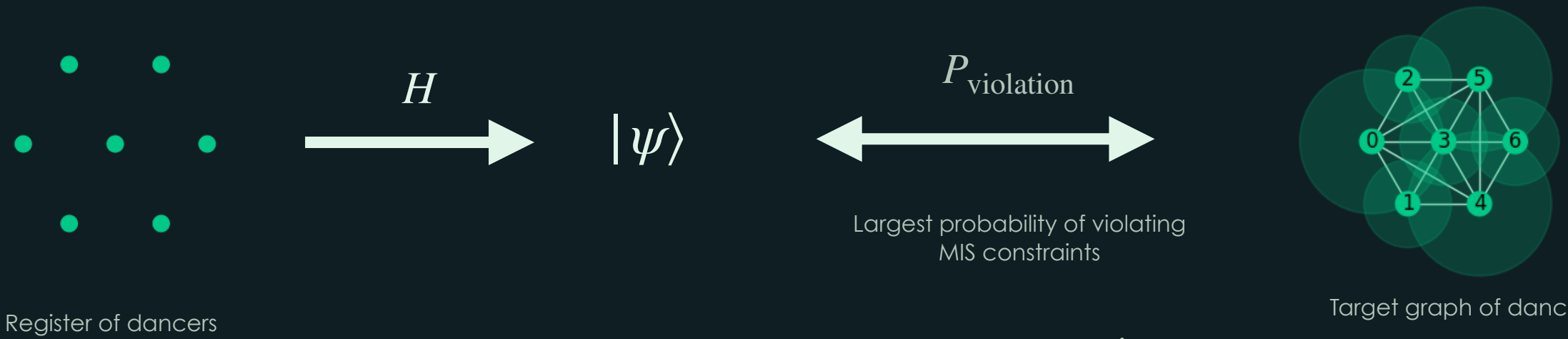
λ



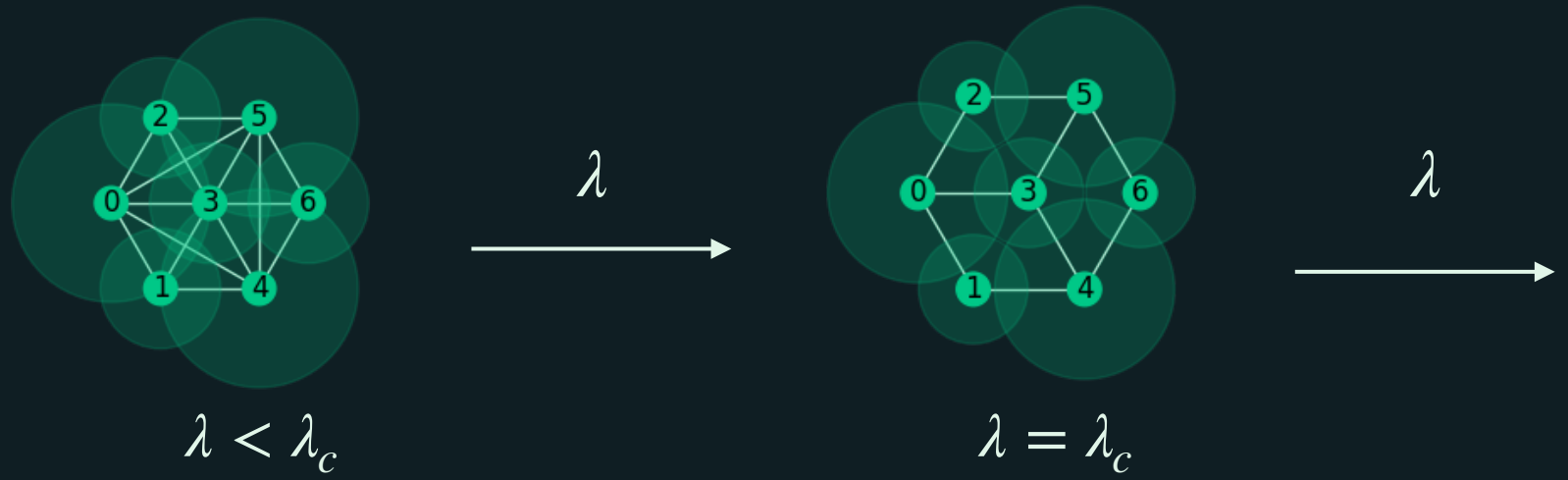
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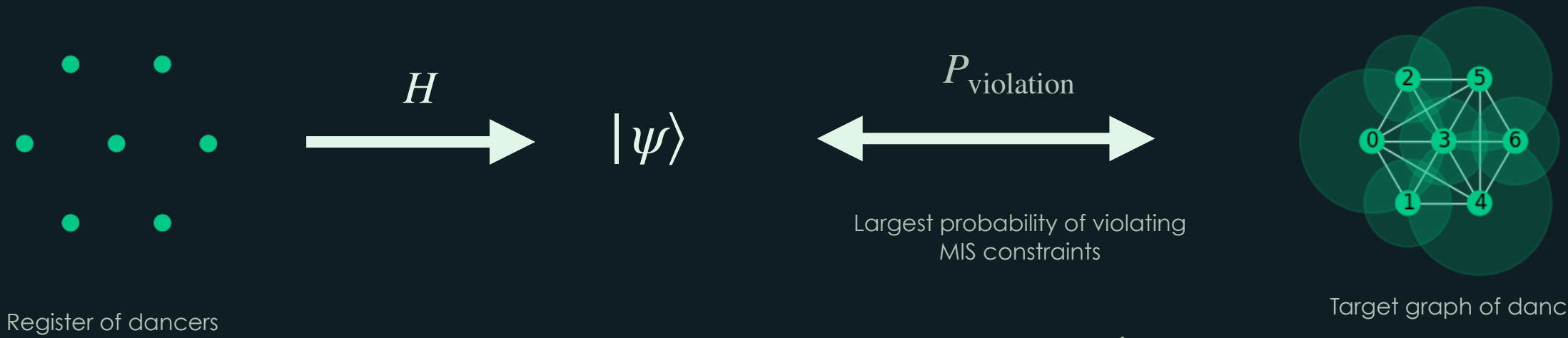


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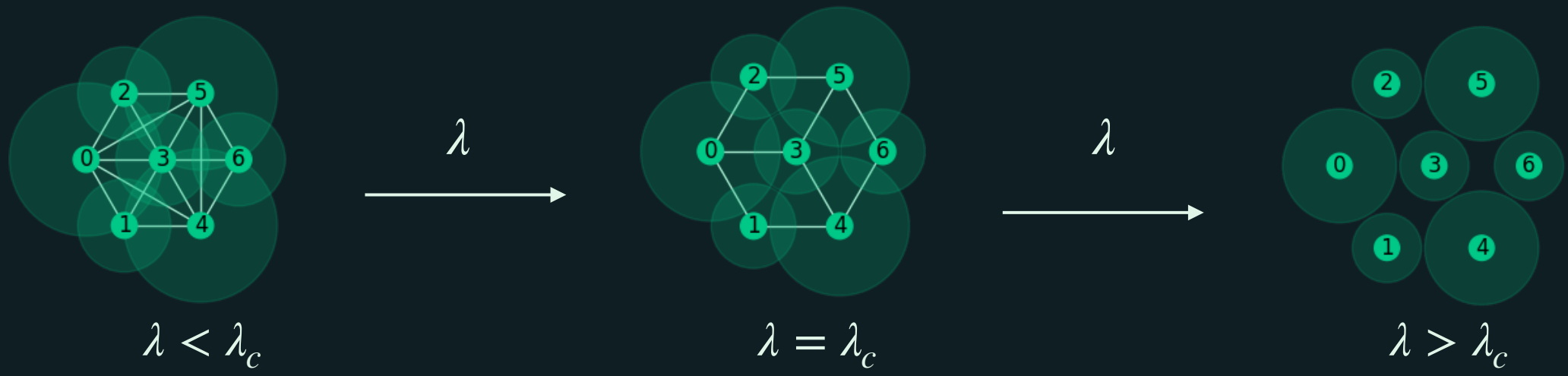


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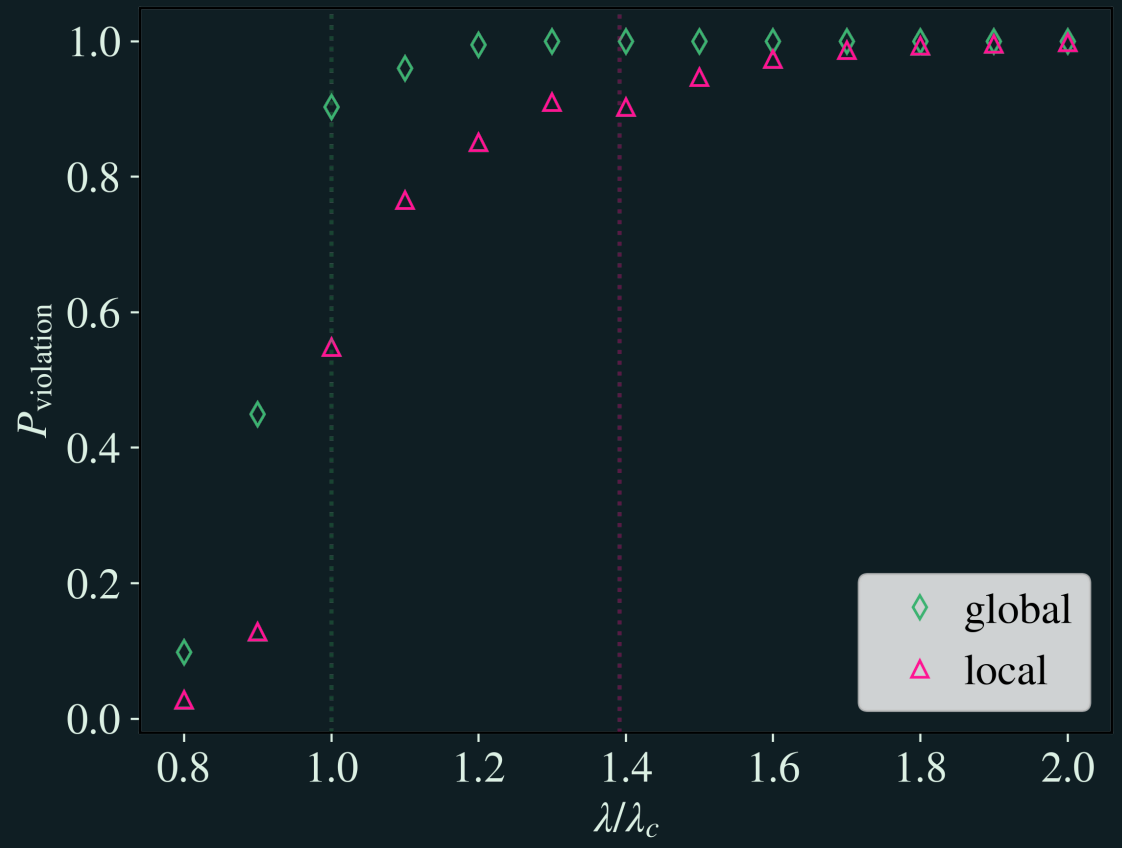


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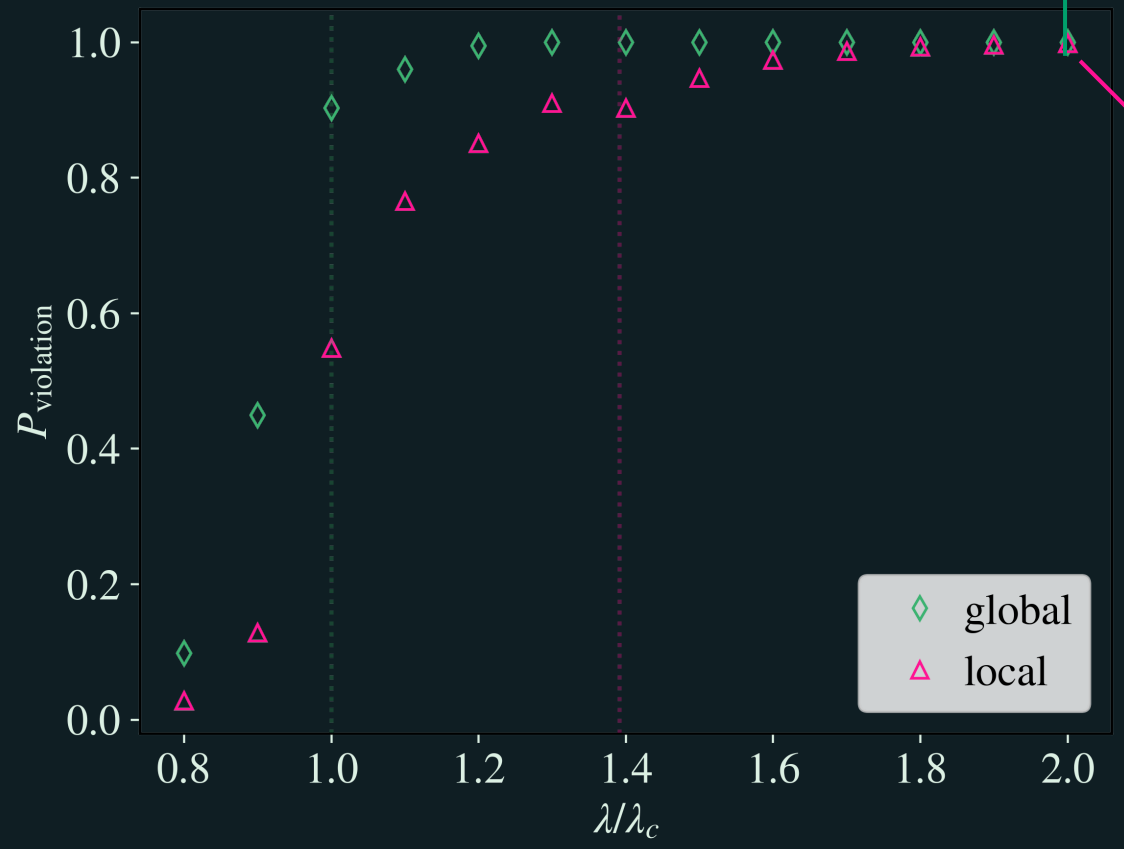


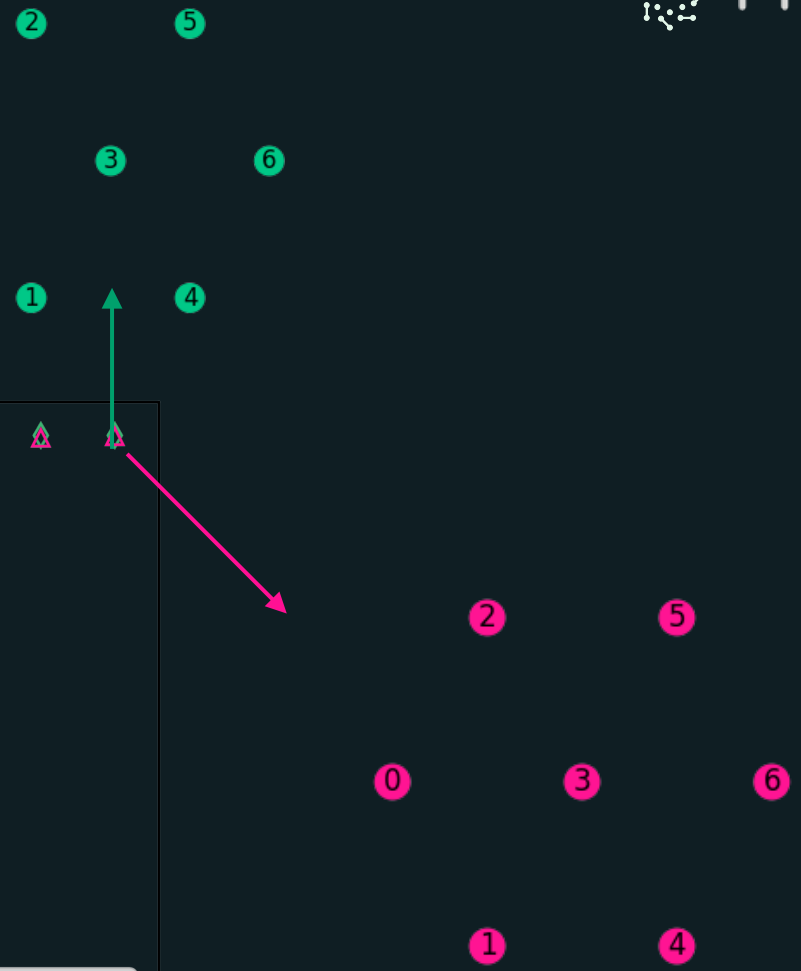
Embedding strategies - Our contribution

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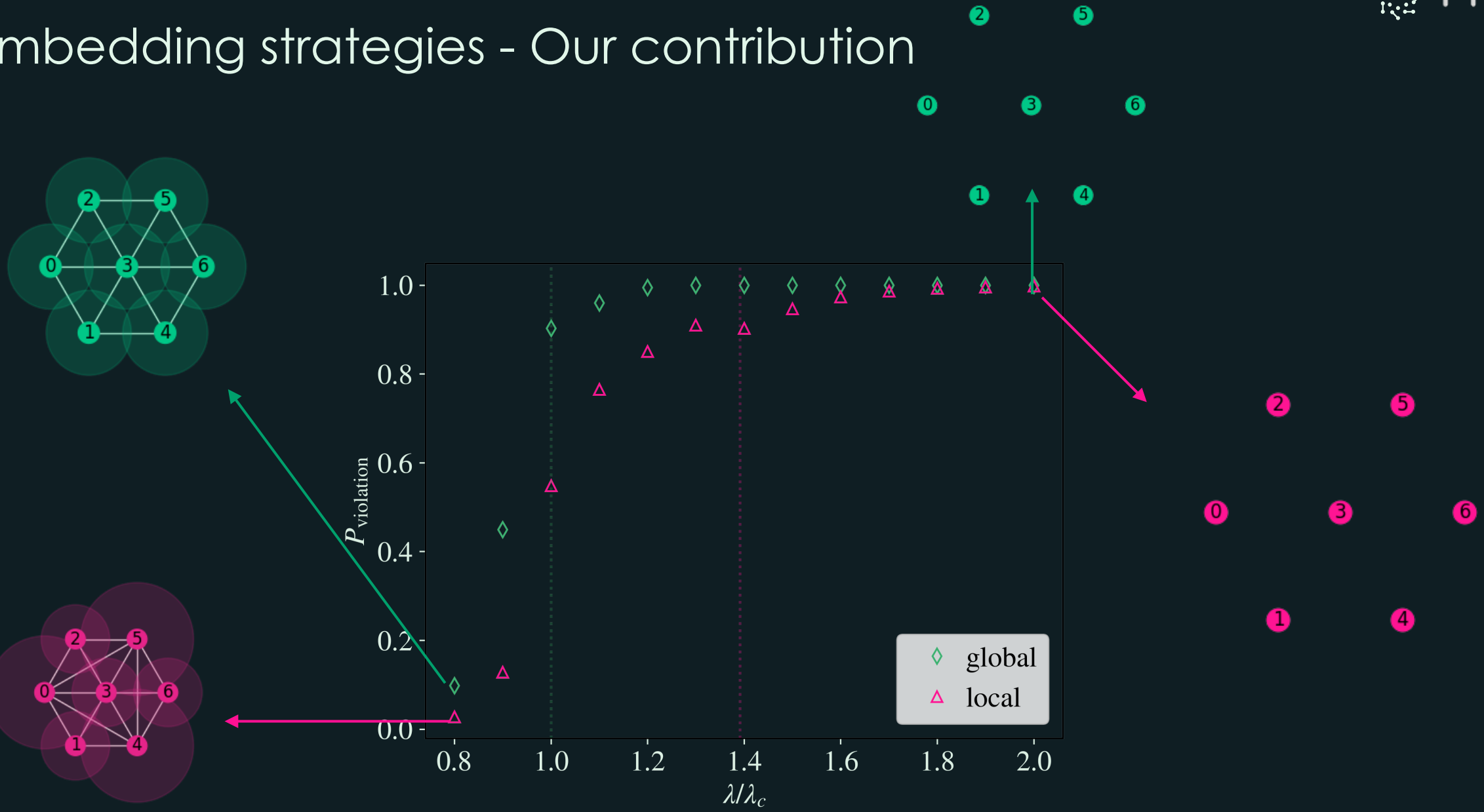


Embedding strategies - Our contribution

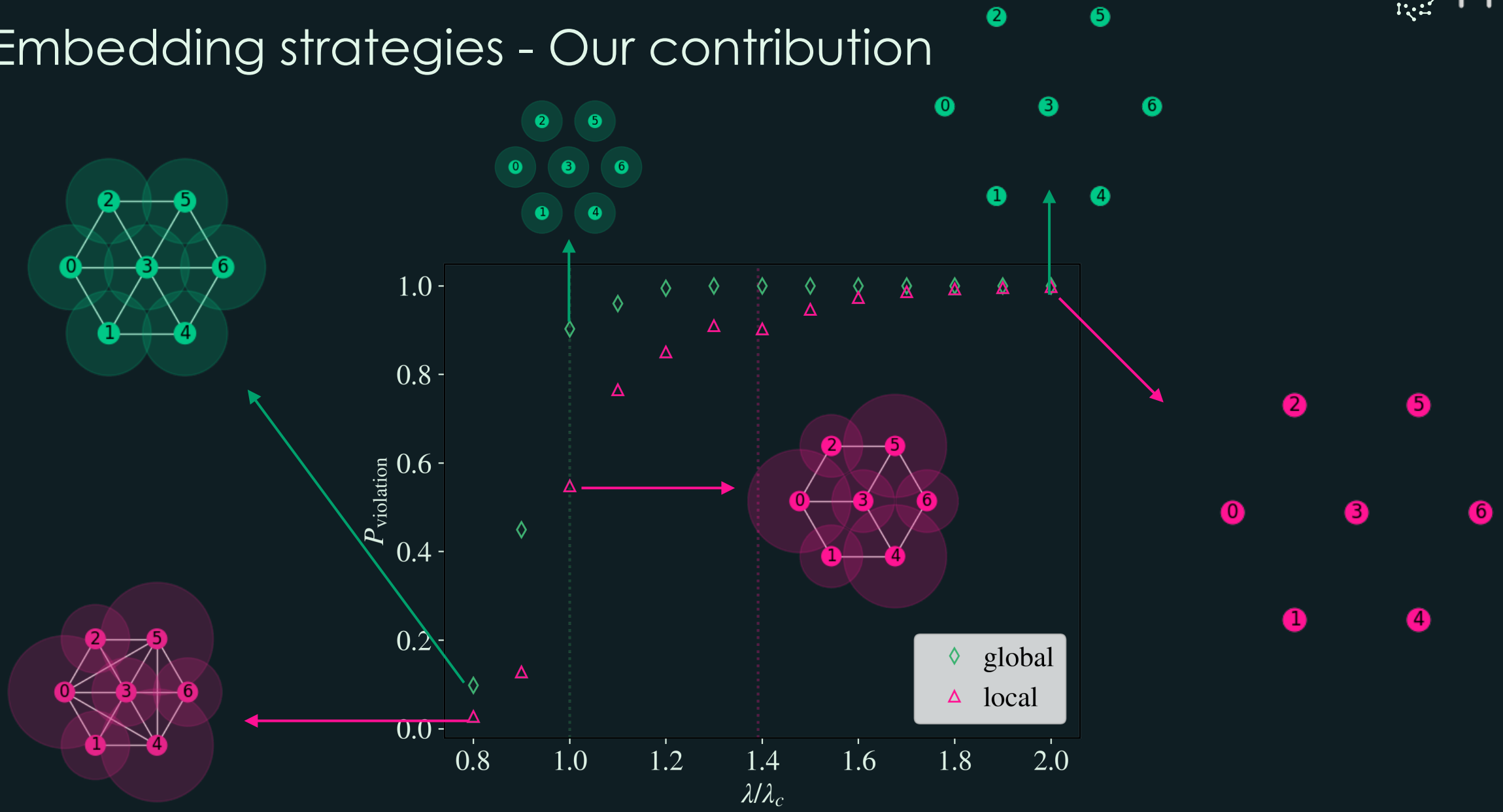




Embedding strategies - Our contribution



Embedding strategies - Our contribution



Pasqal Defining the quantum reality

When does the quantum algorithm work?

[BBA26] Braida Arthur, Bermot Elie, Simon Apers, Log-concavity and tunneling: adiabatic quantum optimization for convex functions (with a spike), 34th Annual European Symposium on Algorithms (ESA 2026), 110: 1-110: 19

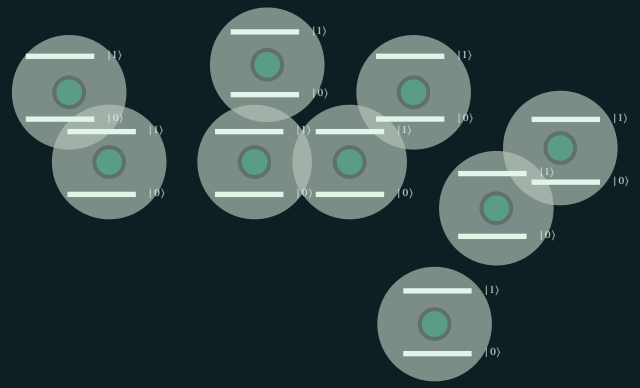
Quantum annealing

Quantum annealing

Start in $|\psi(0)\rangle$ easy ground state of H_0

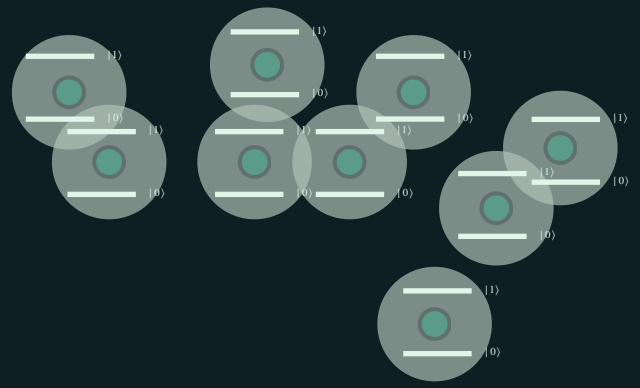
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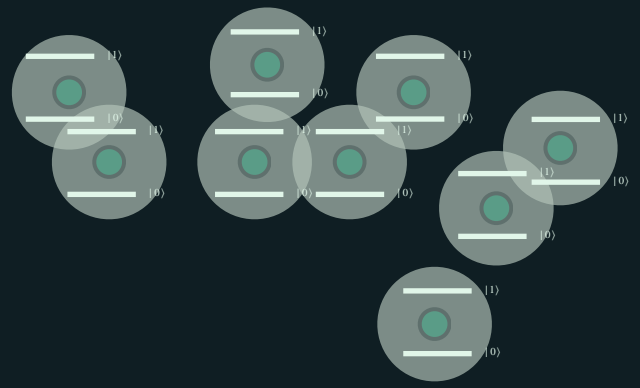
$s = 0$

Every configuration equally likely

Uniform superposition

Quantum annealing

Start in $|\psi(0)\rangle$ easy ground state of H_0



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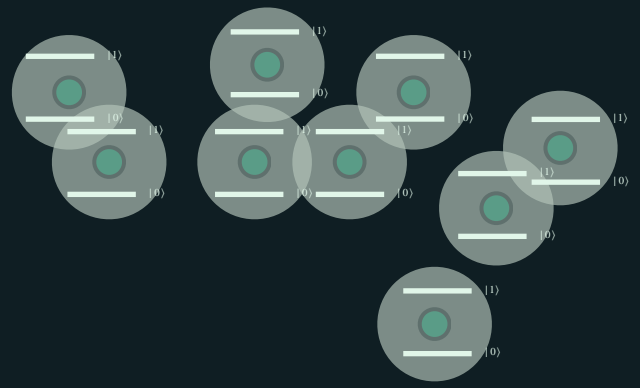
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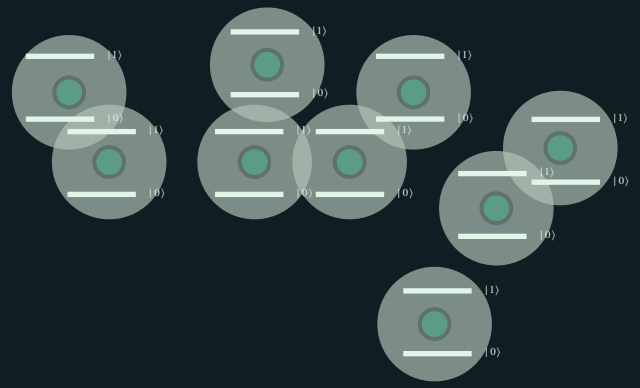
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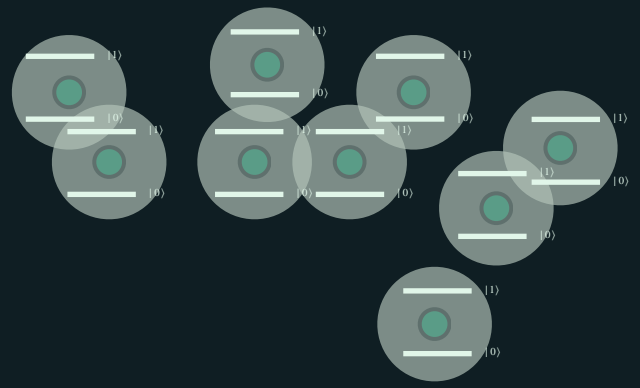
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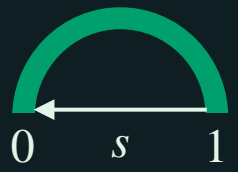
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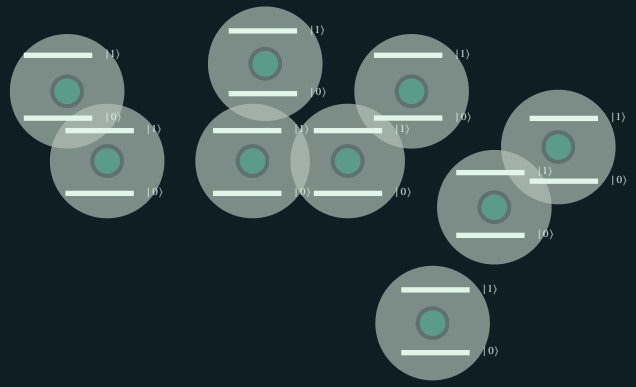


Quantum annealing

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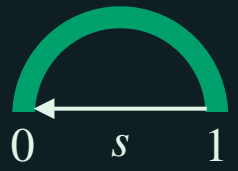




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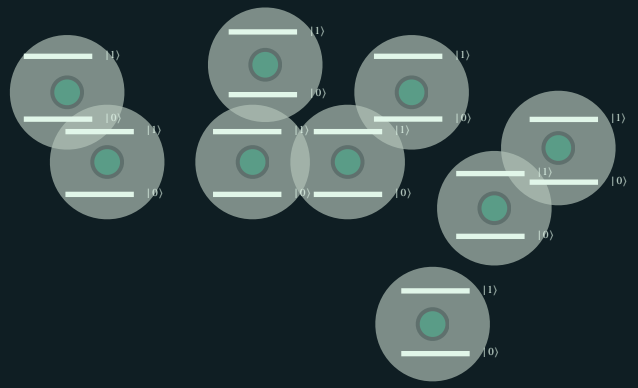


Quantum annealing

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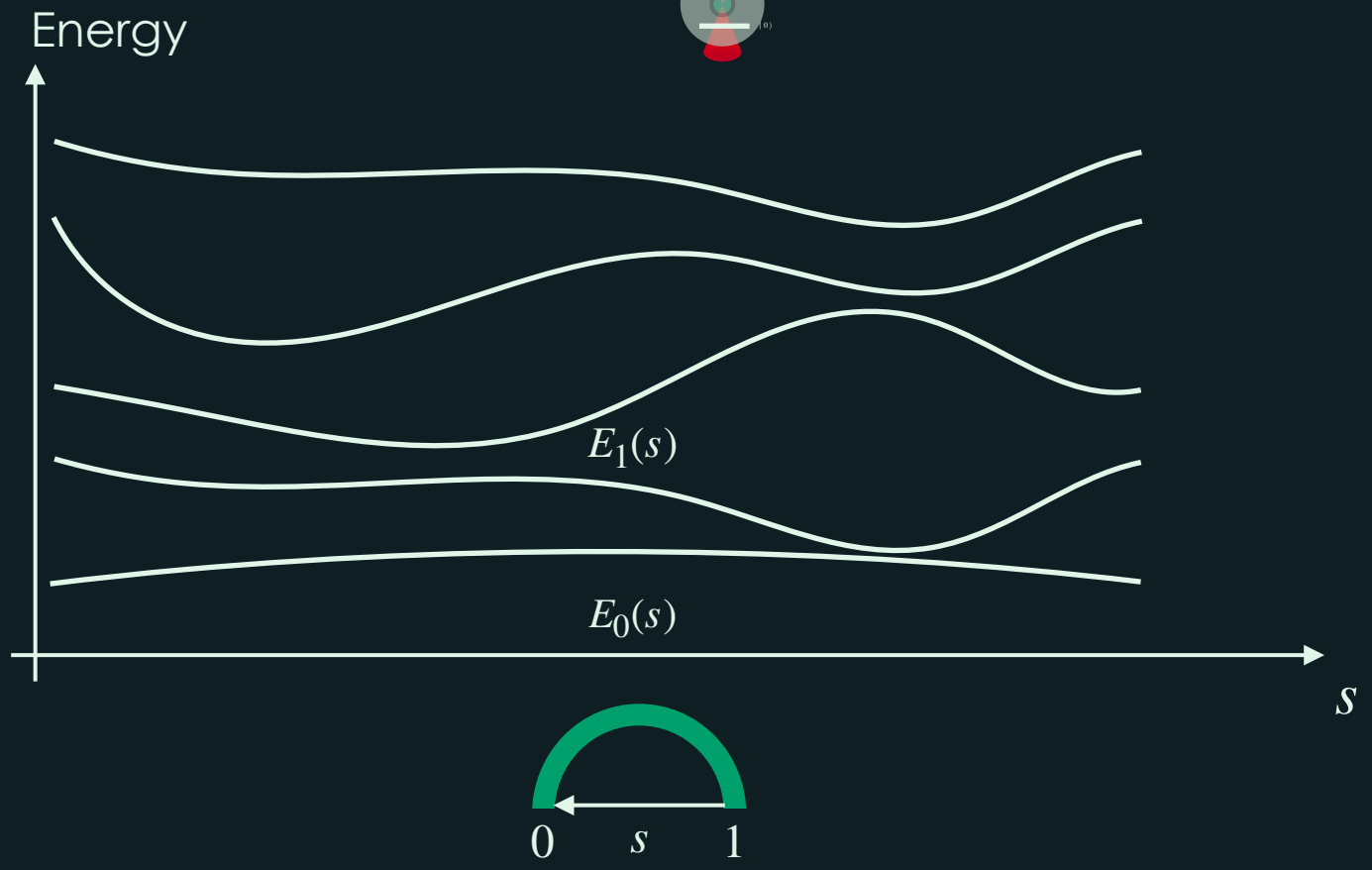




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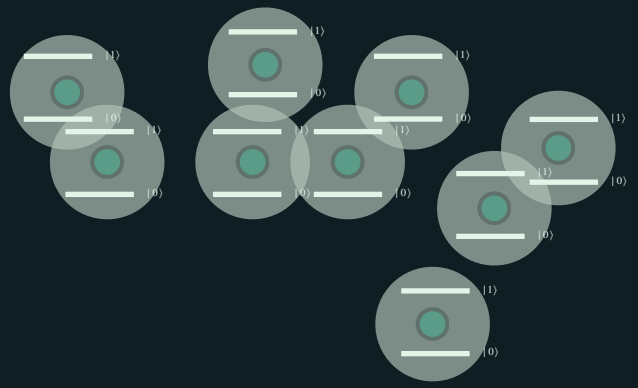


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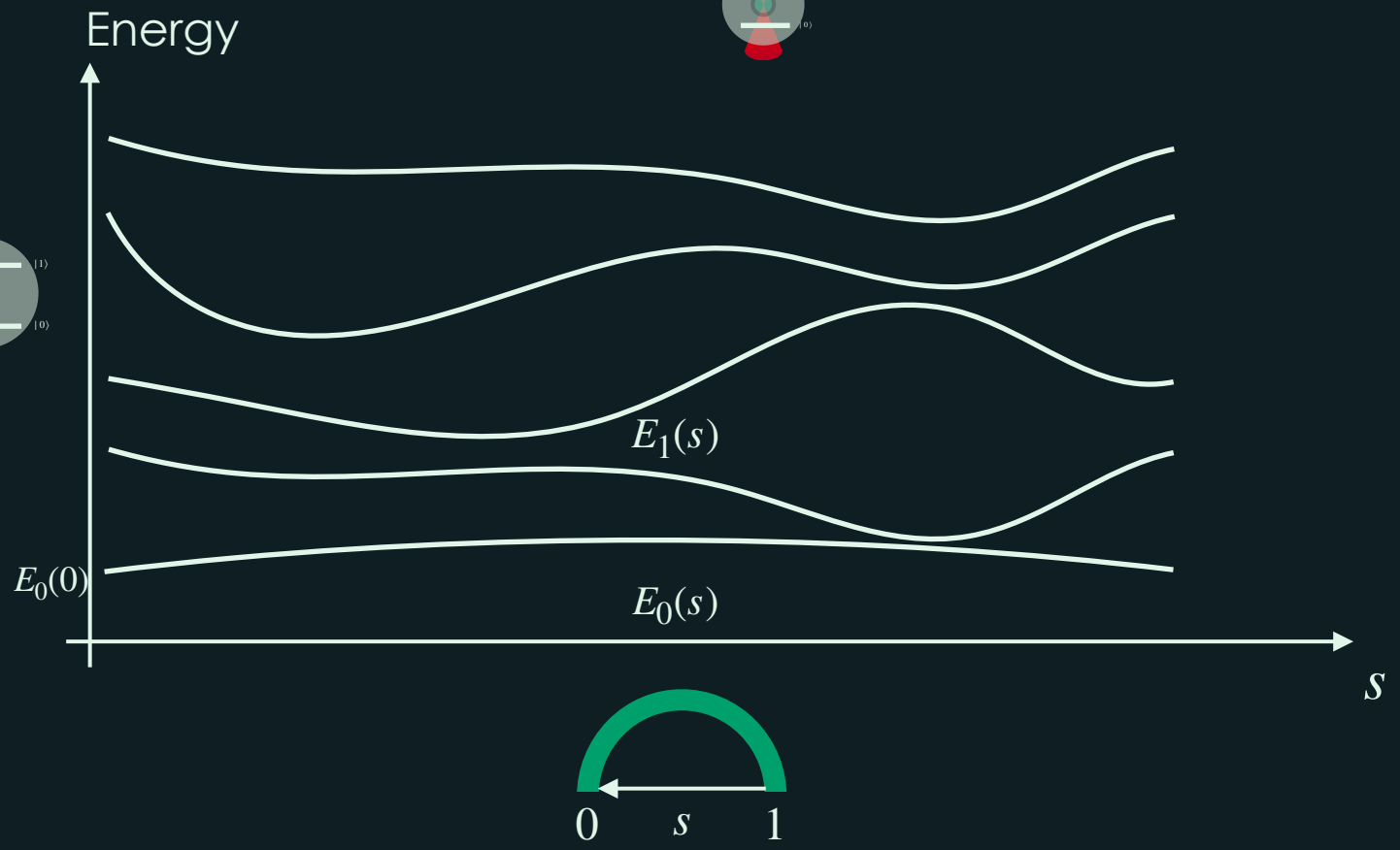




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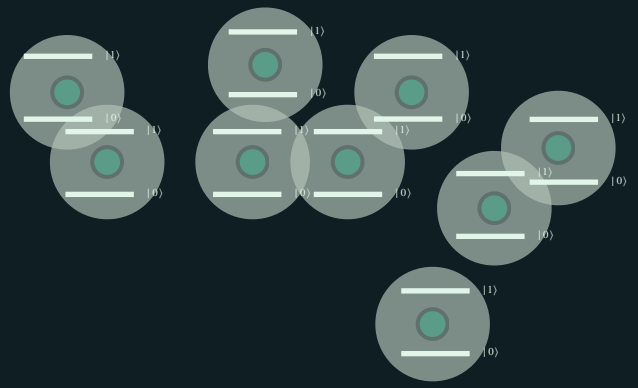


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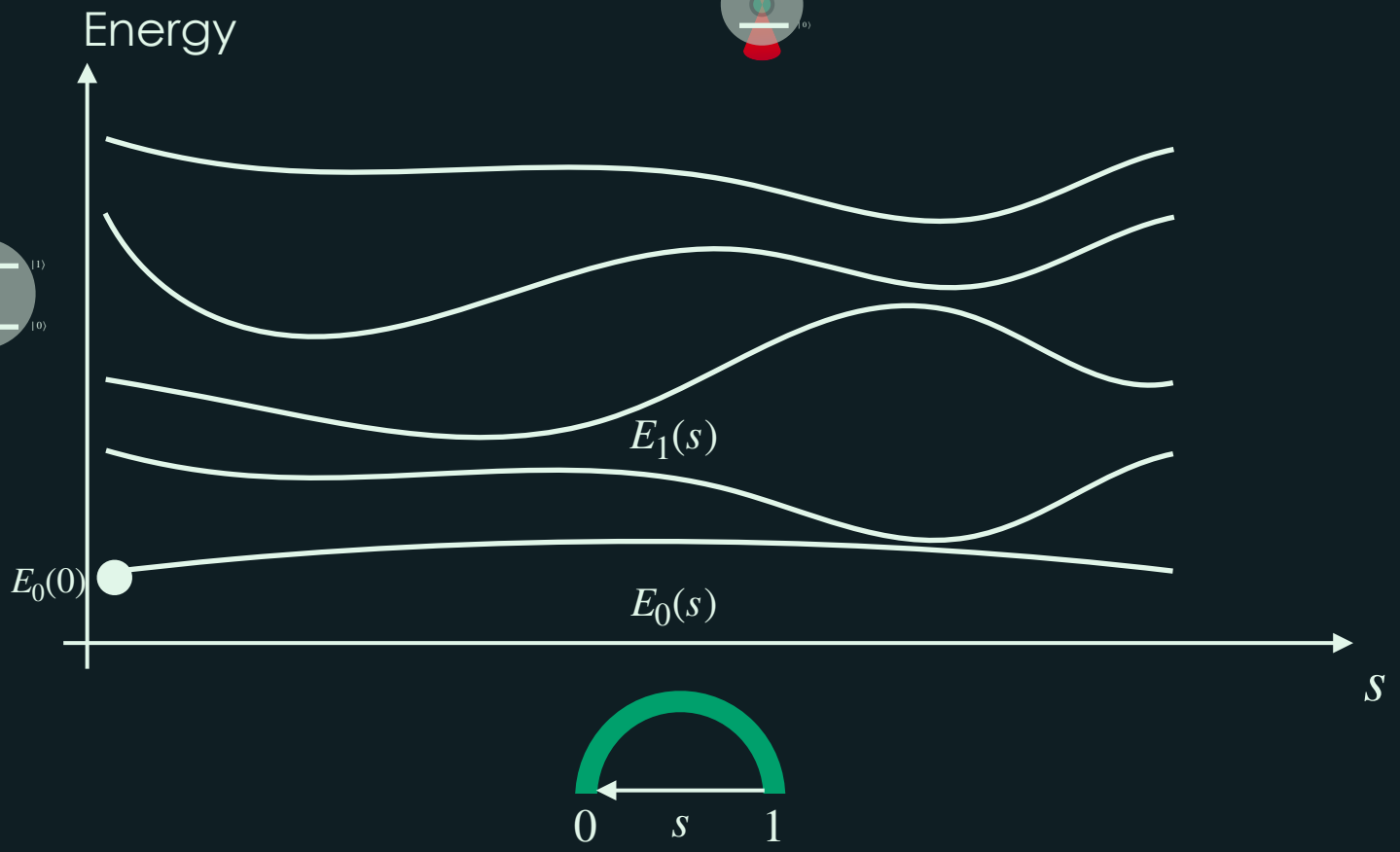




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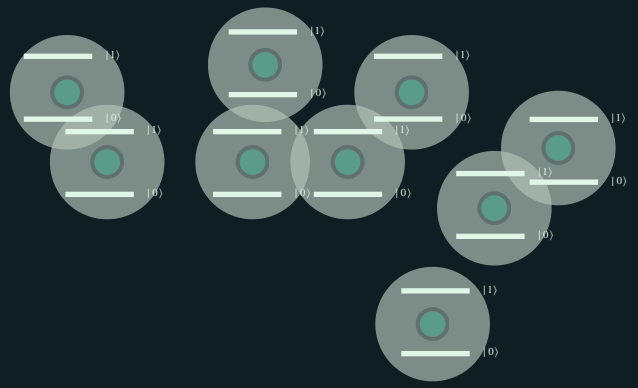


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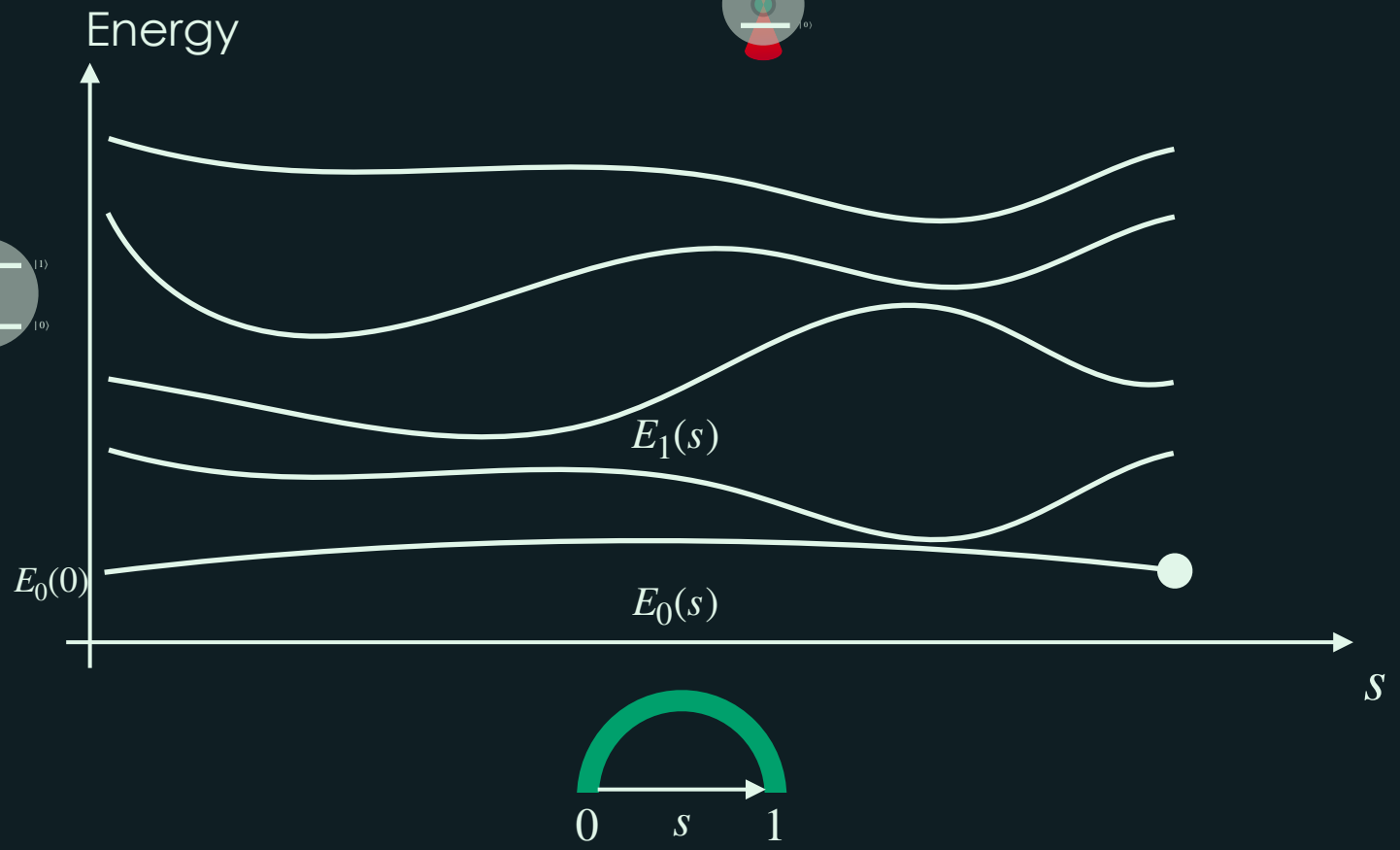




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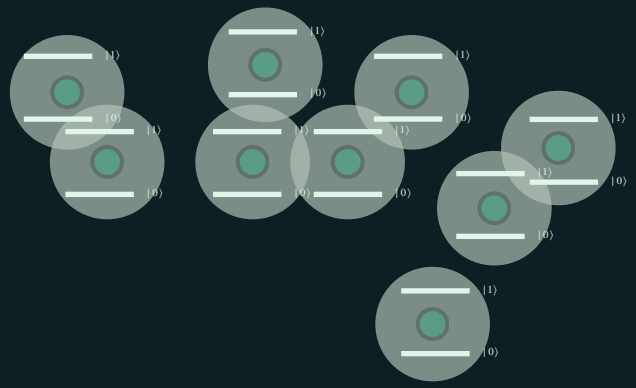
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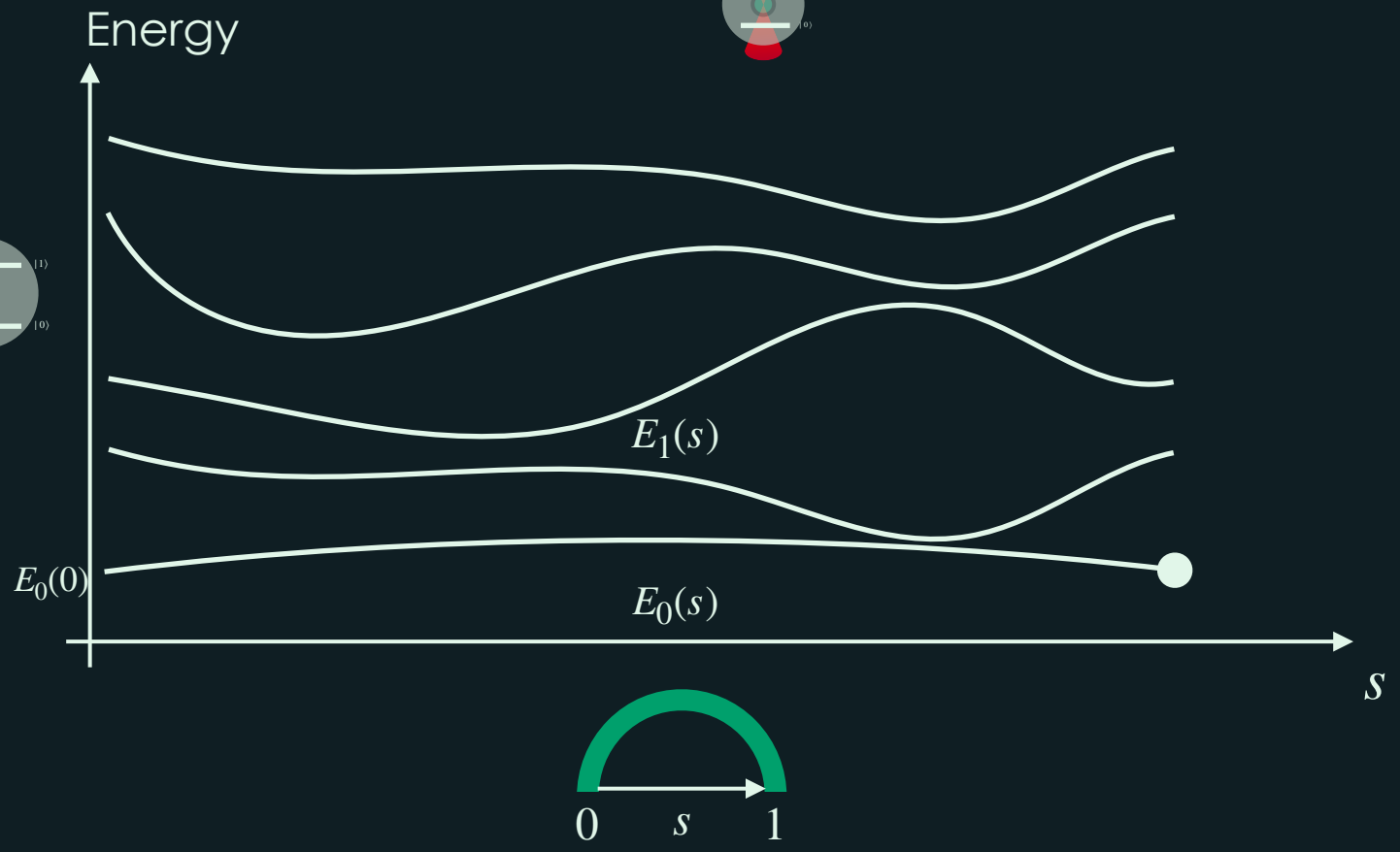
Quantum annealing

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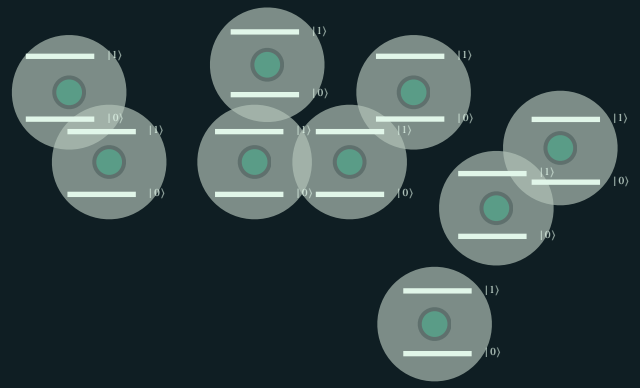
Quantum annealing

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Slow evolution



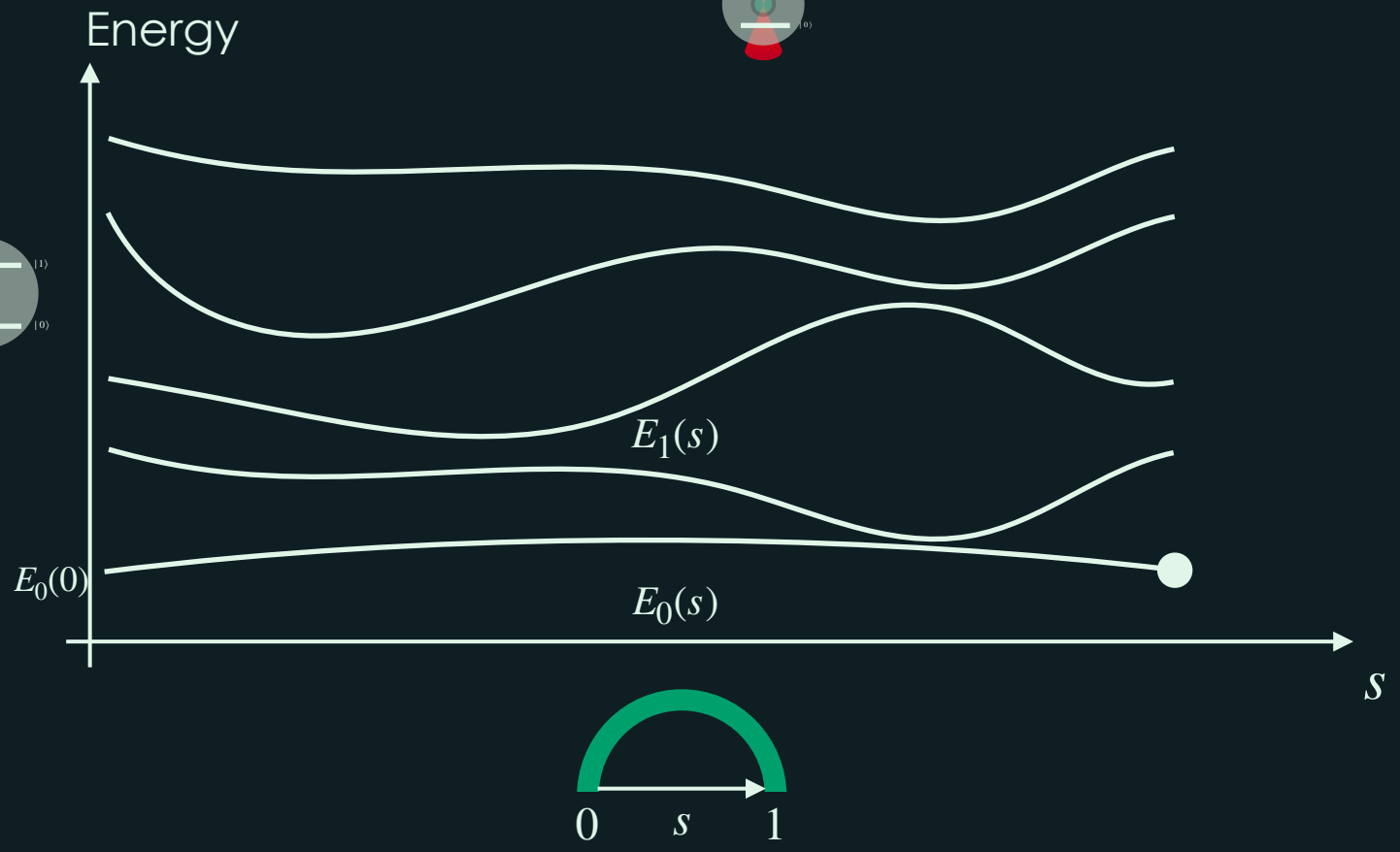
Final state $|\psi(1)\rangle$ close to the ground state of H_c



$s = 0$

Every configuration equally likely

Uniform superposition



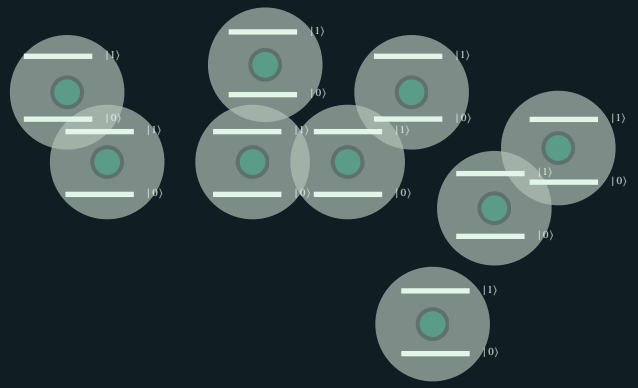
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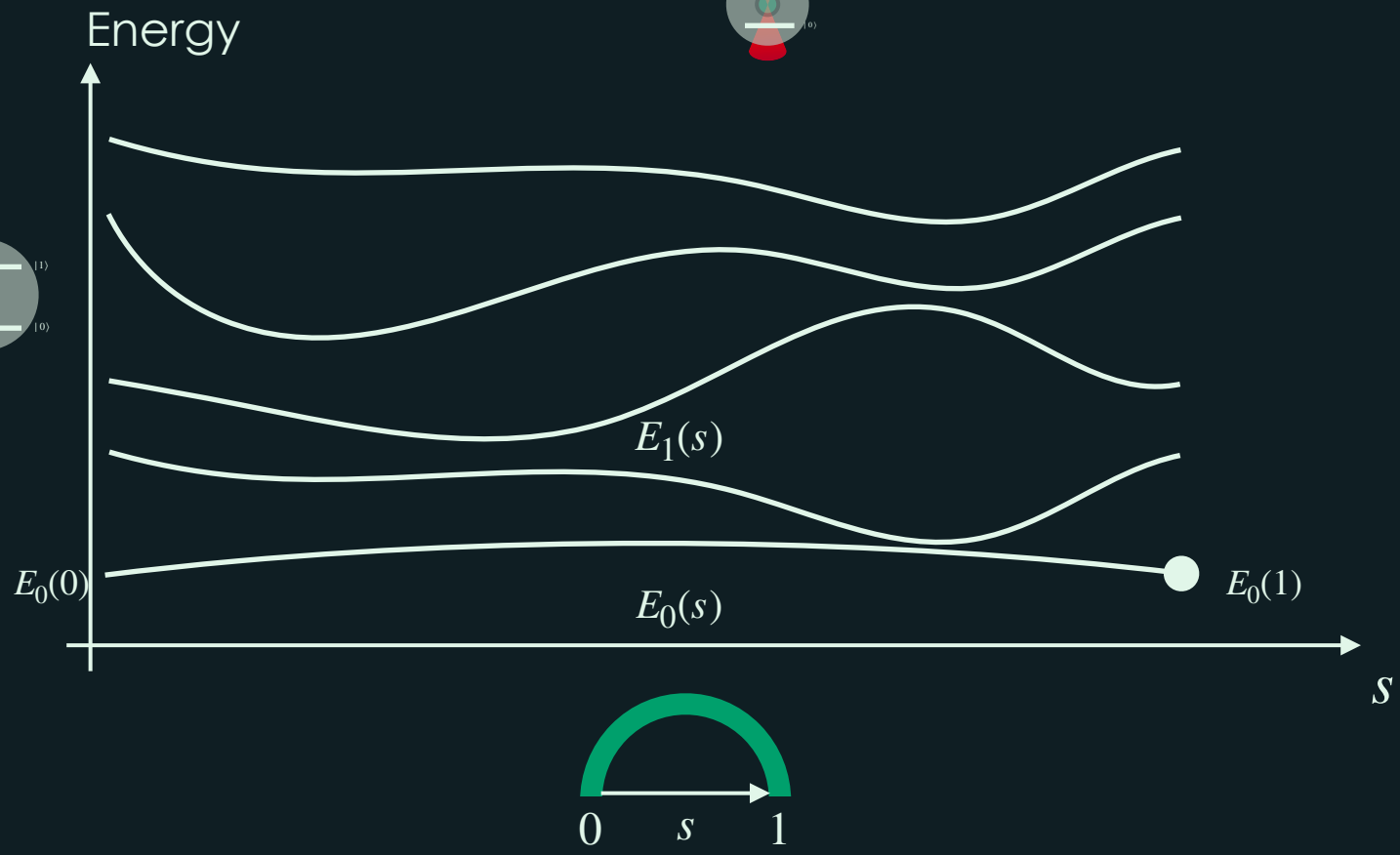
Slow evolution



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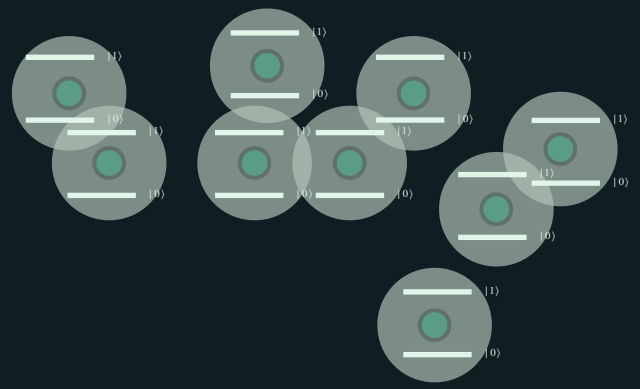
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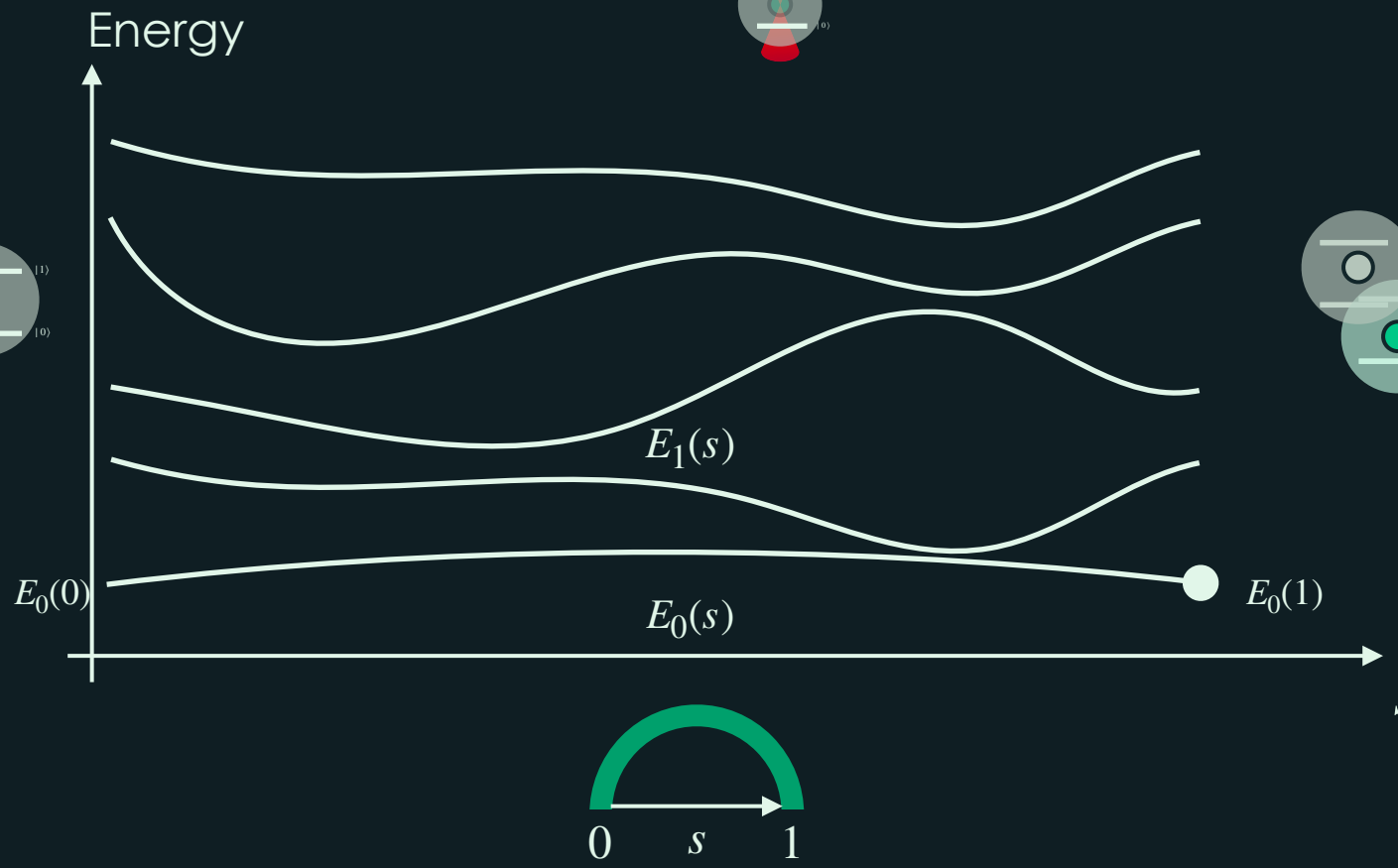
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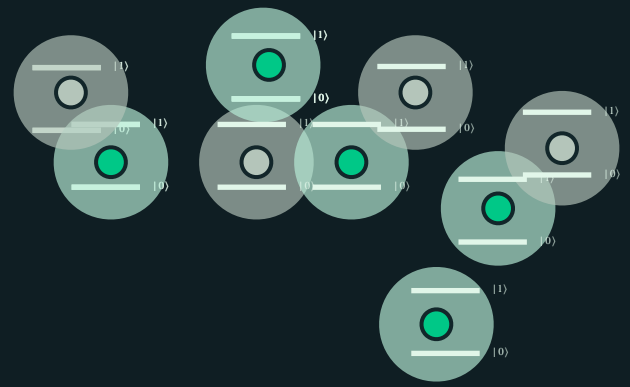


Final state $|\psi(1)\rangle$ close to the ground state of H_c



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Every configuration equally likely
Uniform superposition





$s = 1$
Solution of the optimization problems
Which (quantum) dancer can spin

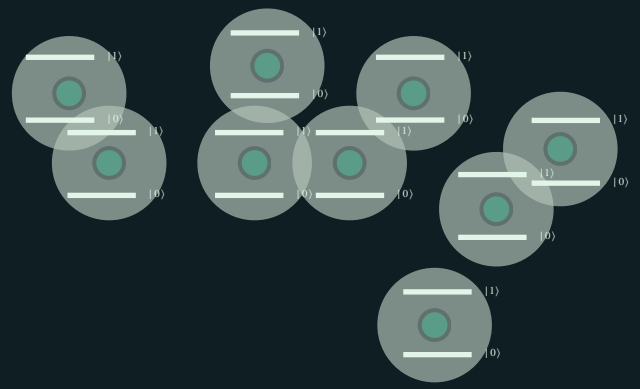
Quantum annealing

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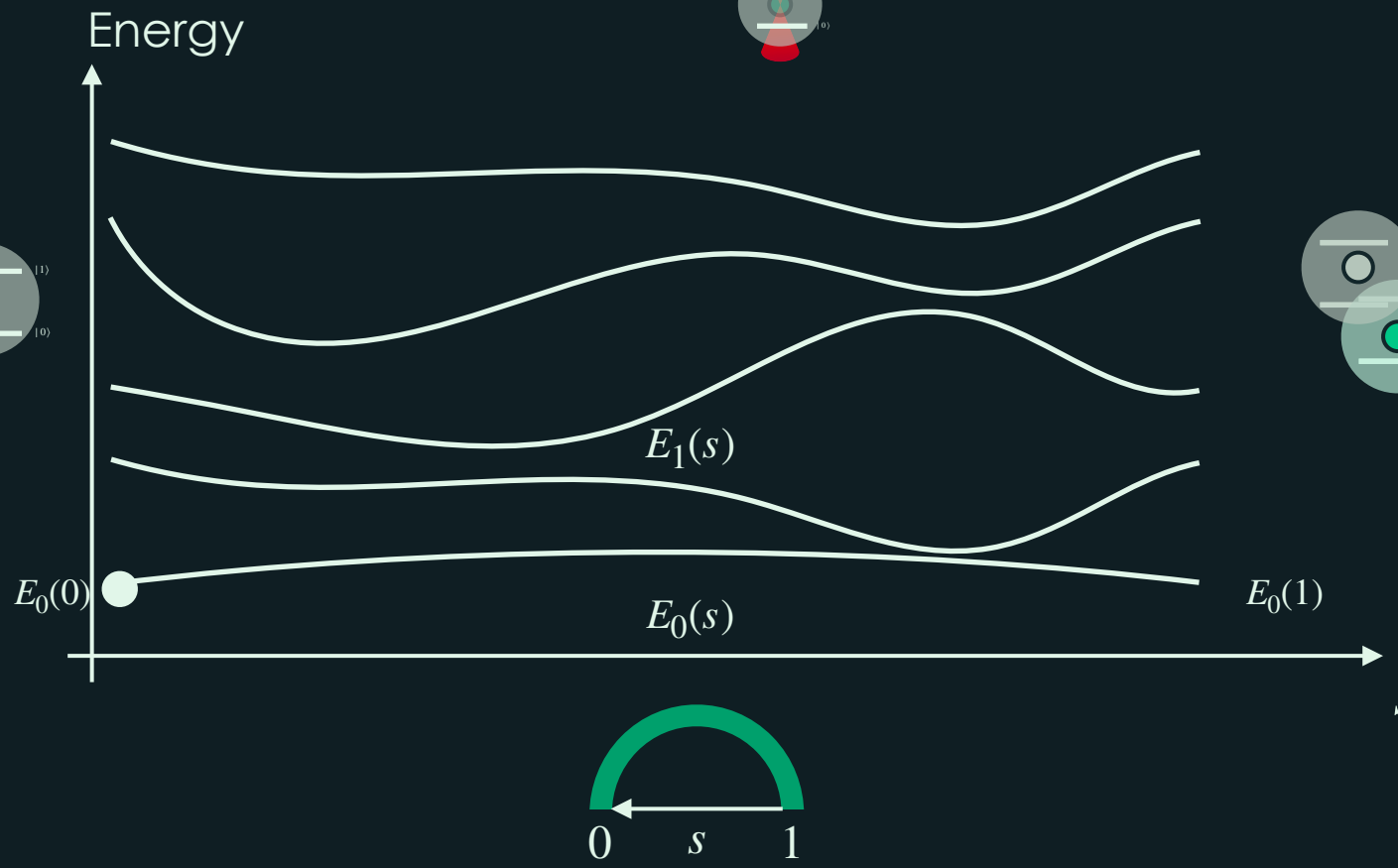
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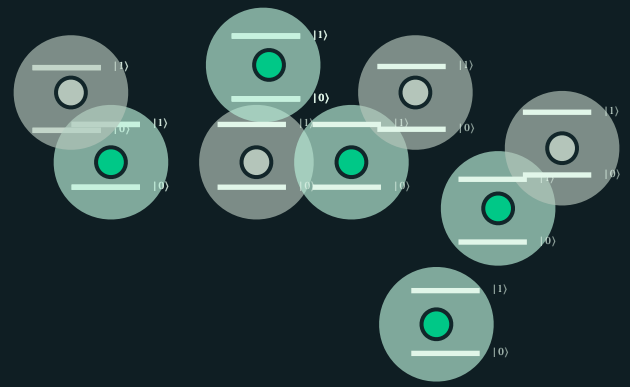


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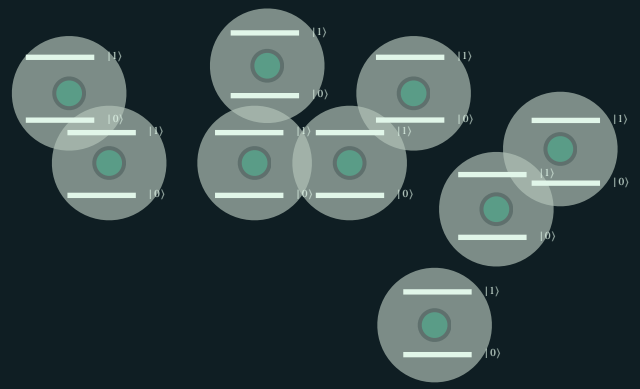
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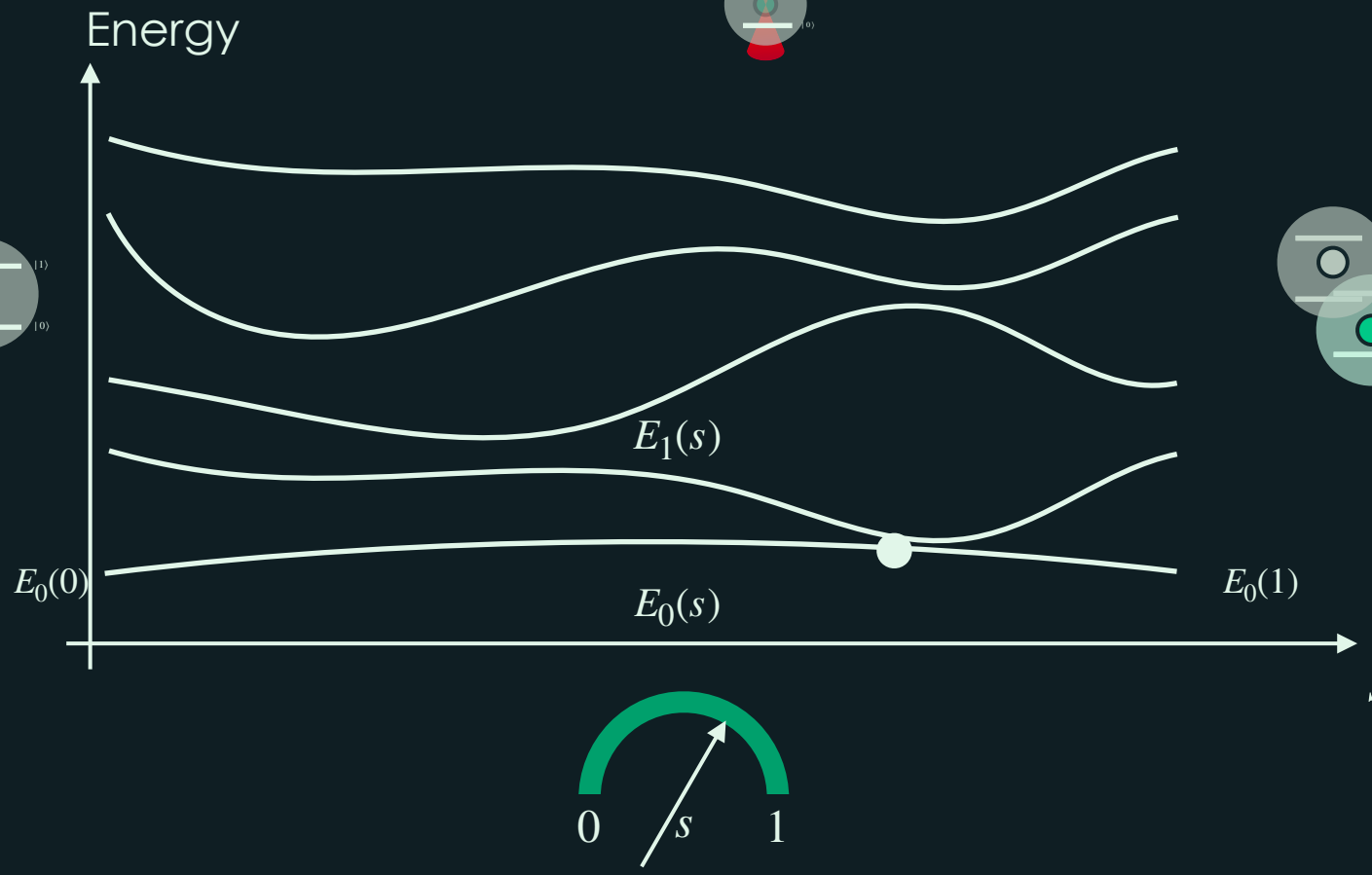
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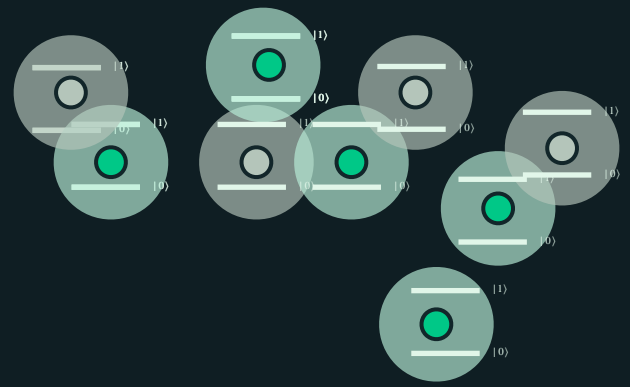


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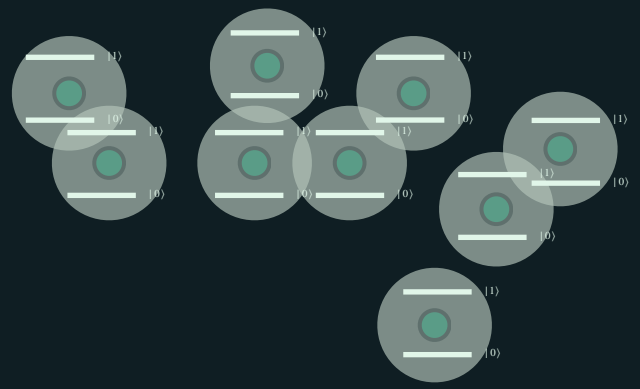
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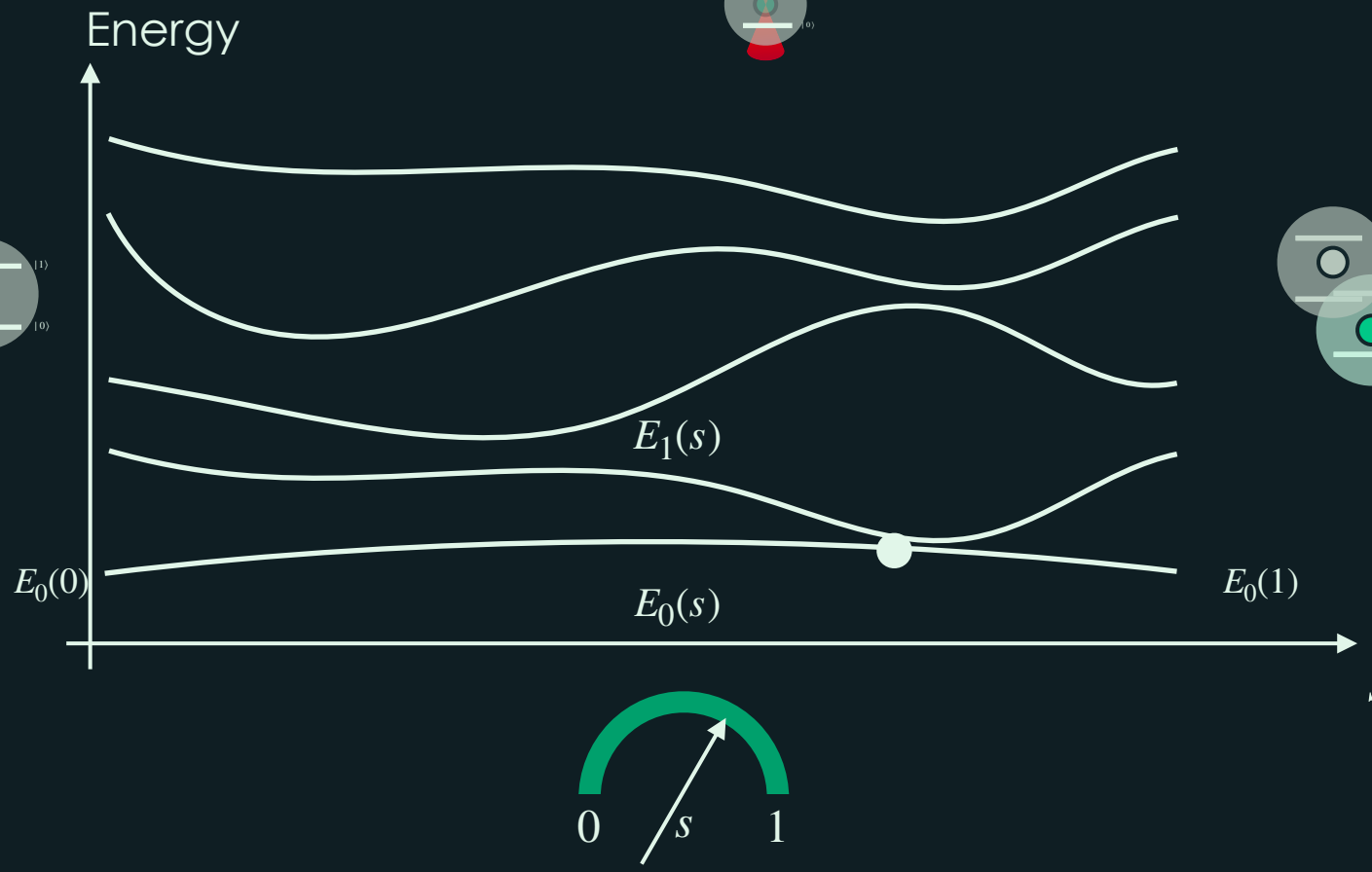
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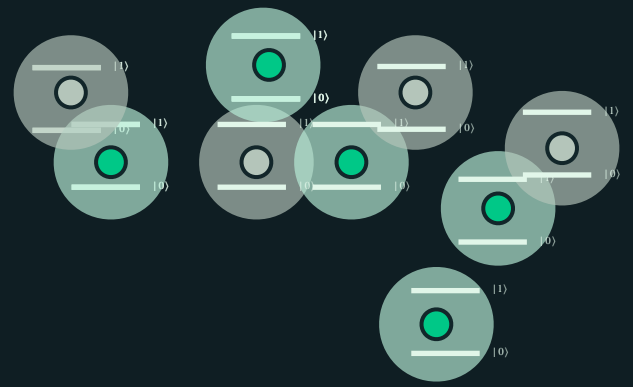


Final state $|\psi(1)\rangle$ close to the ground state of H_c



$s = 0$
Every configuration equally likely
Uniform superposition





$s = 1$
Solution of the optimization problems
Which (quantum) dancer can spin

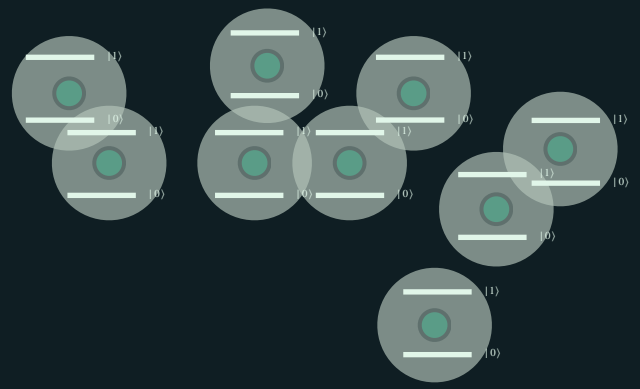
Quantum annealing

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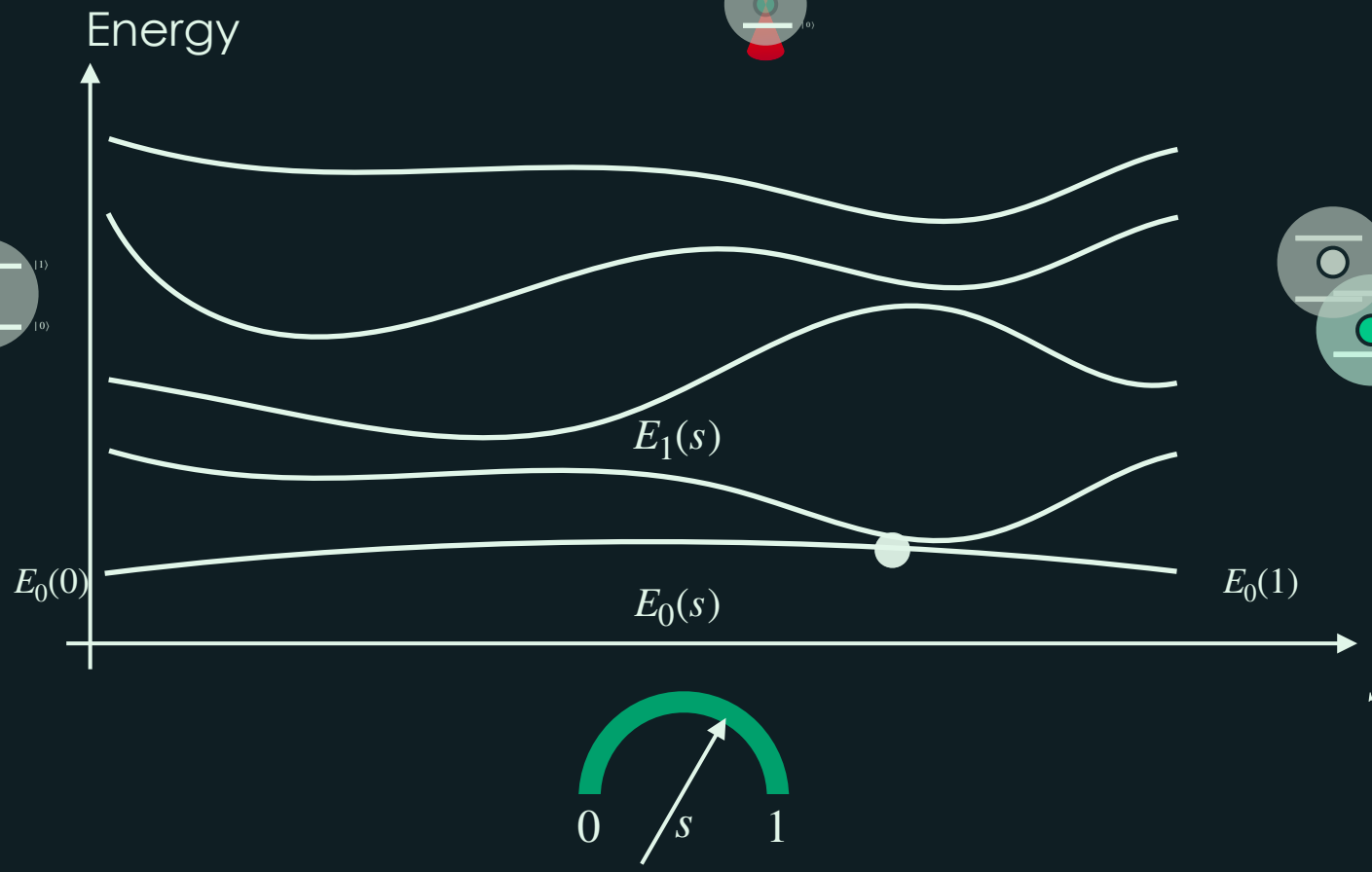
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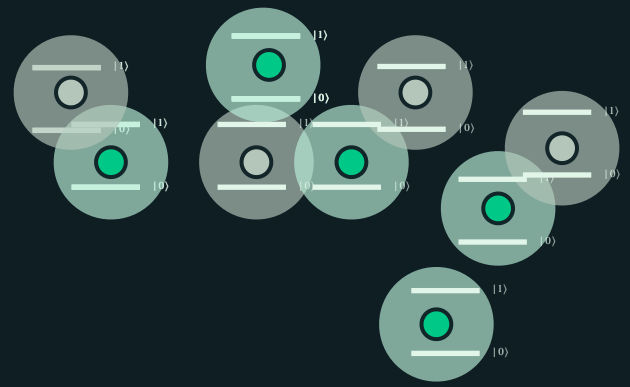


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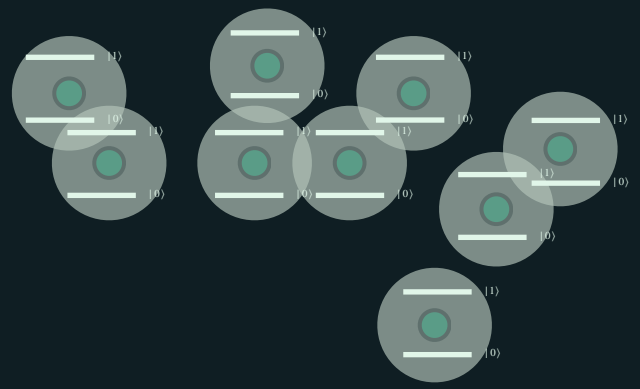
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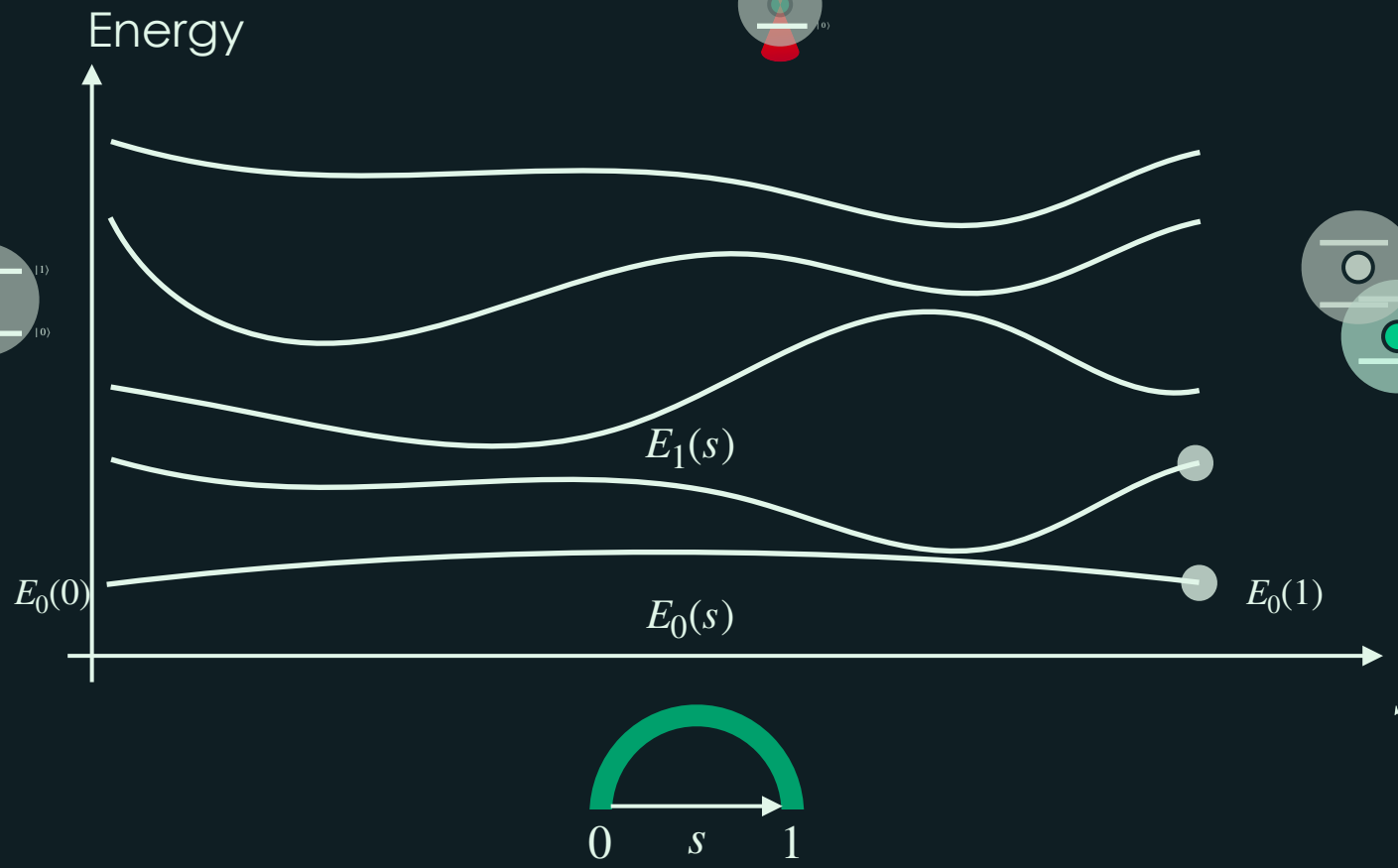
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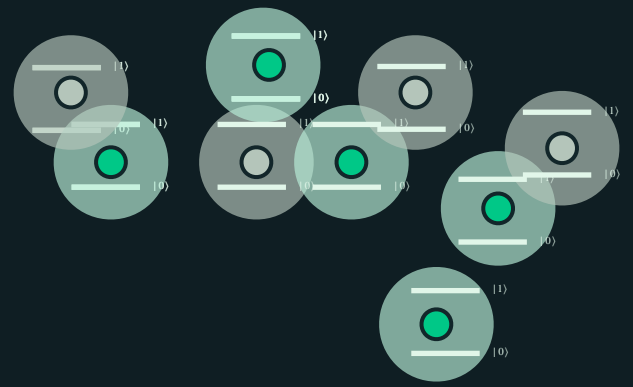


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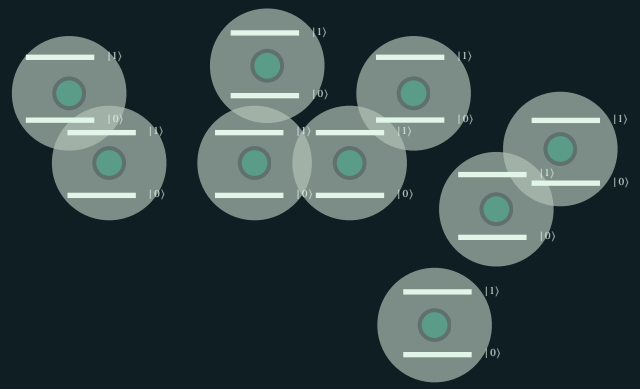
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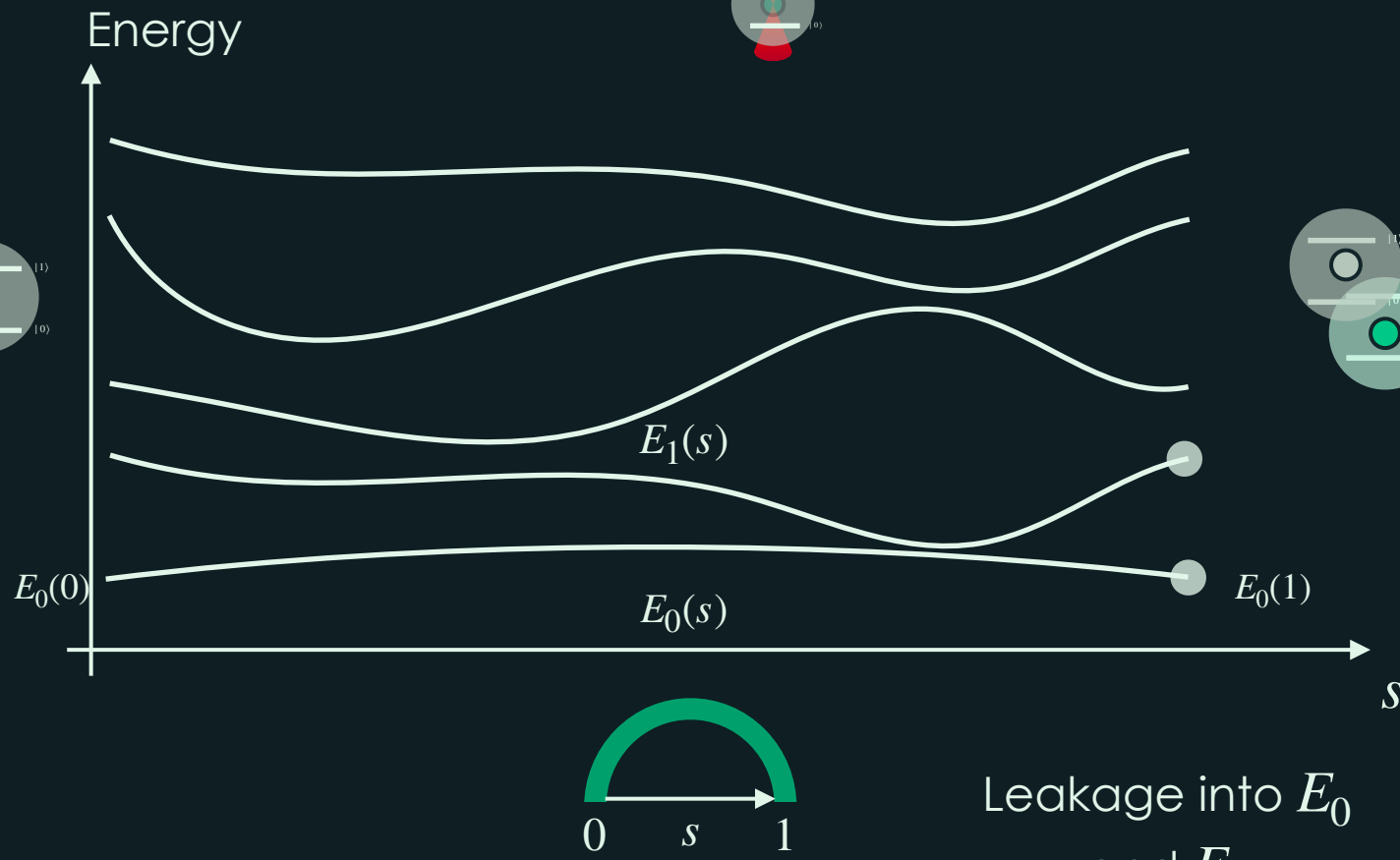
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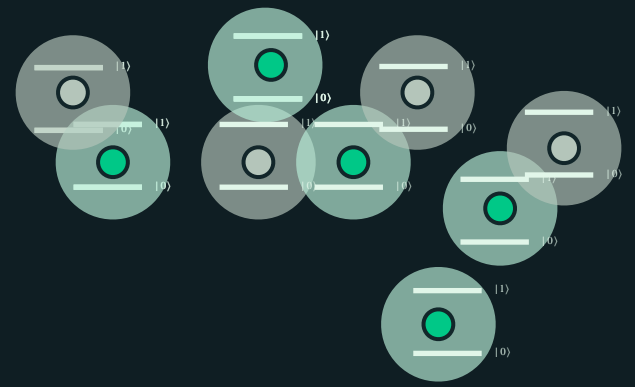
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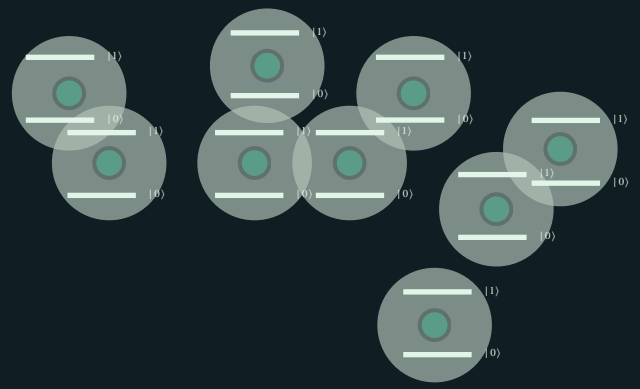
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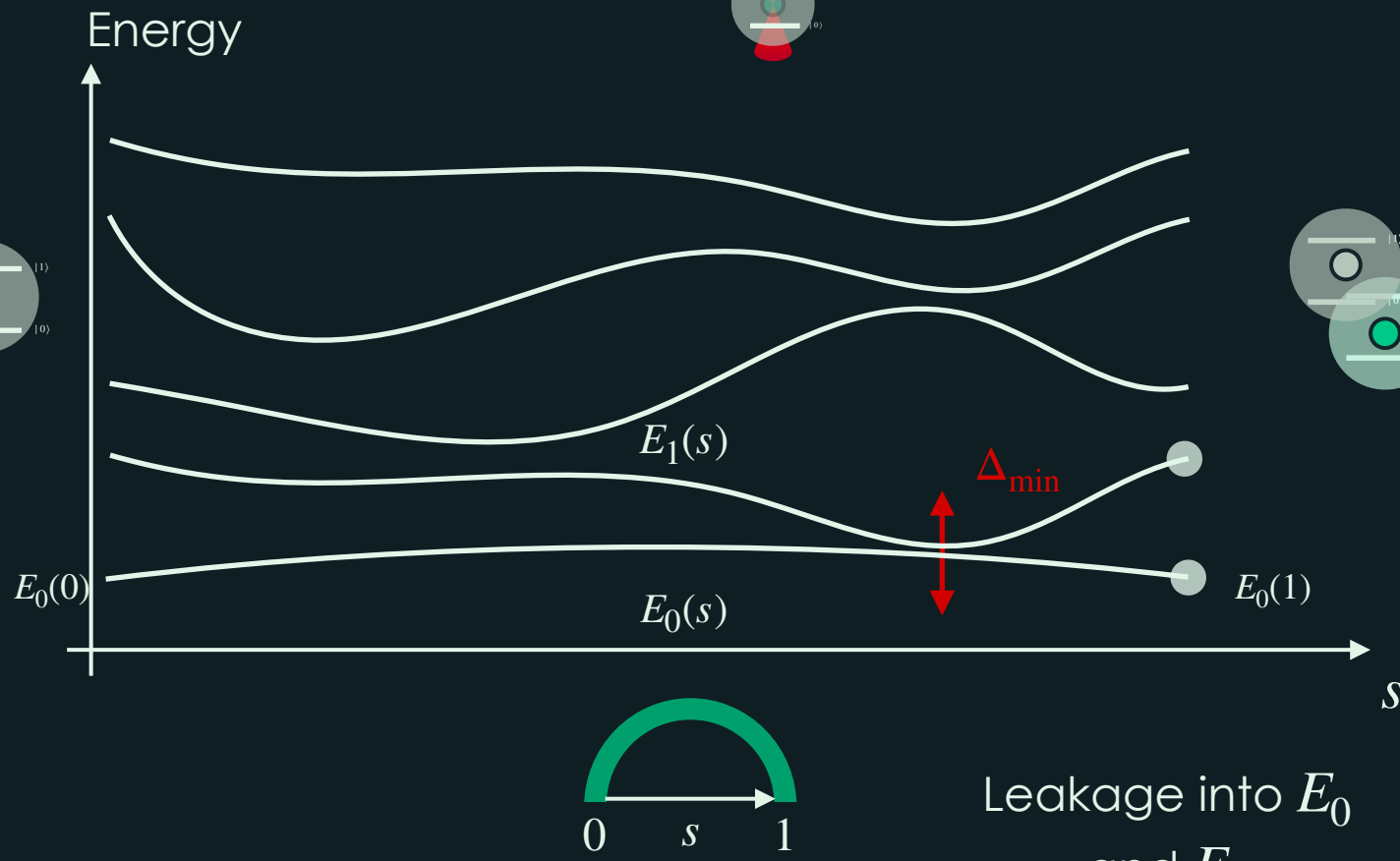
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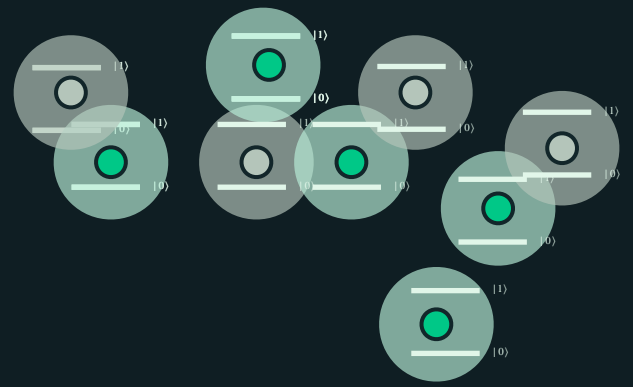
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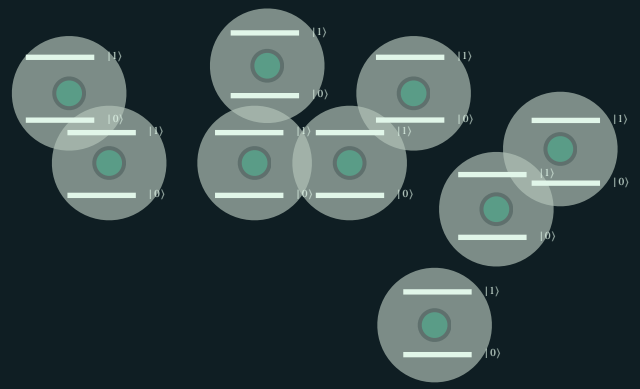
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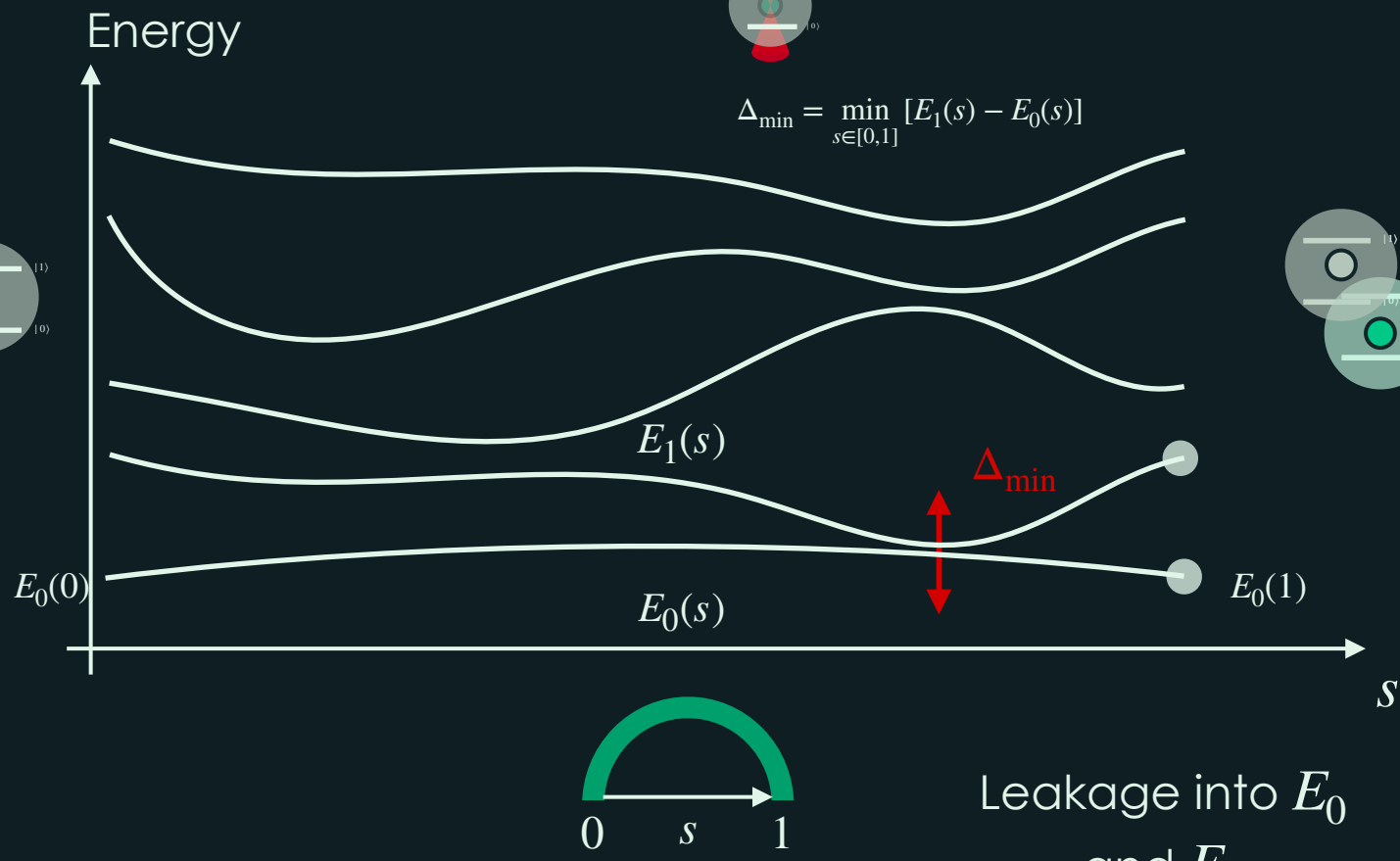
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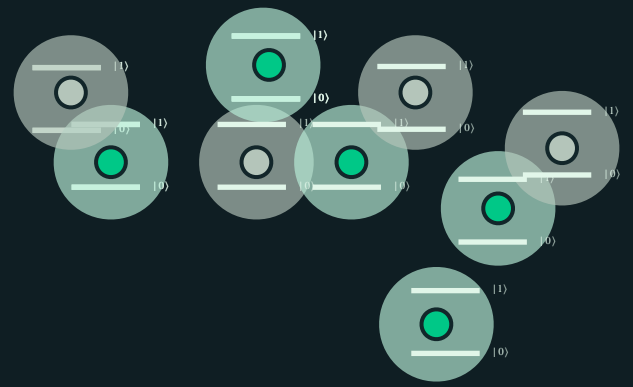


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Leakage into E_0 and E_1

Quantum annealing

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Slow evolution



Final state $|\psi(1)\rangle$ close to the ground state of H_c

Energy

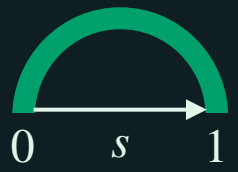
$$\Delta_{\min} = \min_{s\in[0,1]} [E_1(s) - E_0(s)]$$

Adiabatic theorem: Speed at which we change s depends on $\Delta_{\min}$: the smaller $\Delta_{\min}$ is , the slower we should change s

$E_0(0)$ $E_0(s)$ $E_0(1)$ $\Delta_{\min}$ s

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Leakage into E_0 and E_1

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Final state $|\psi(1)\rangle$ close to the ground state of H_c

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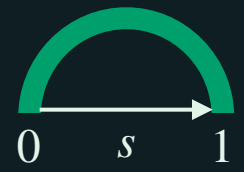
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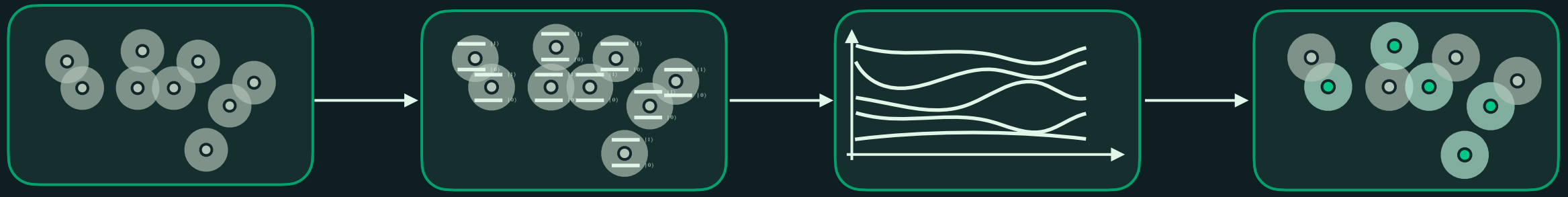
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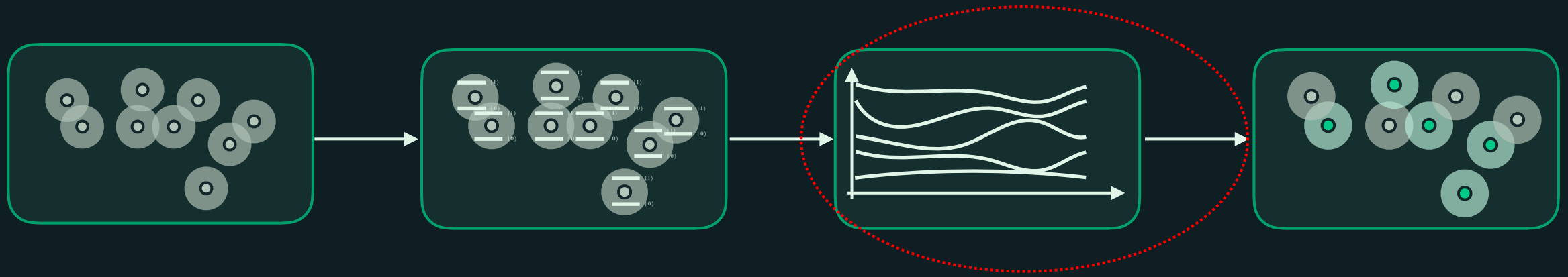
Leakage into E_0 and E_1

Physicist:
$T = O(\Delta_{\min}^{-3})$

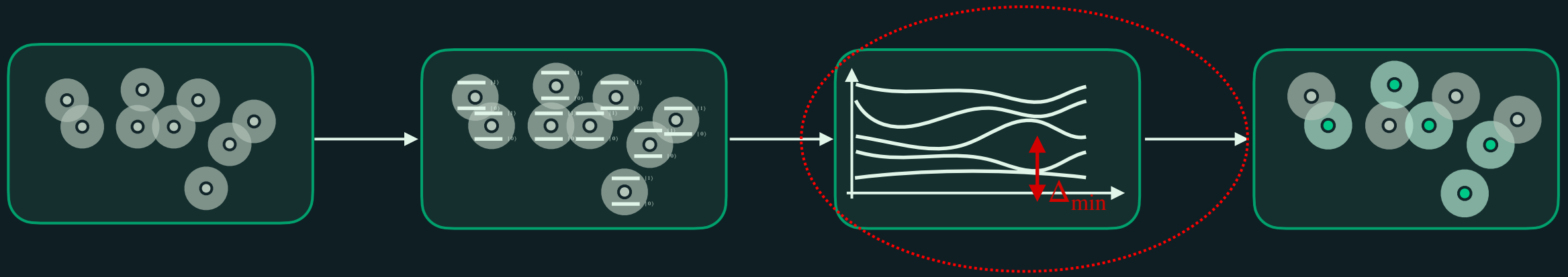
Quantum annealing



Quantum annealing

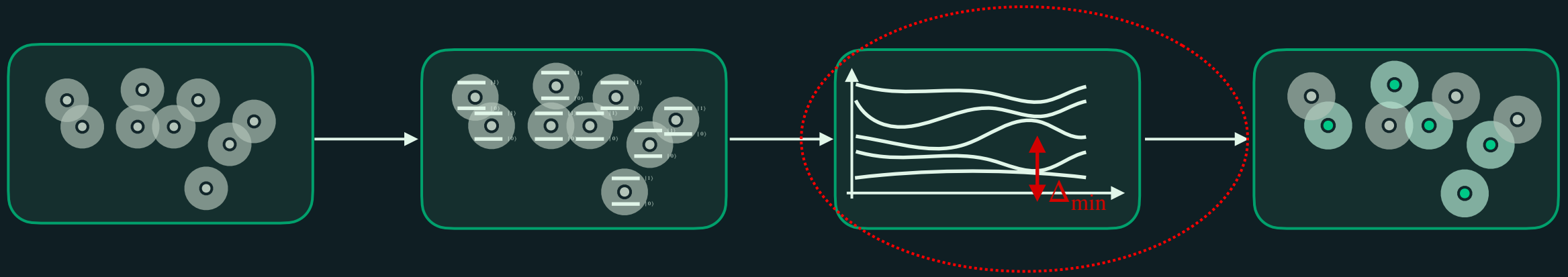


Quantum annealing



Success depends on Δ_{min}

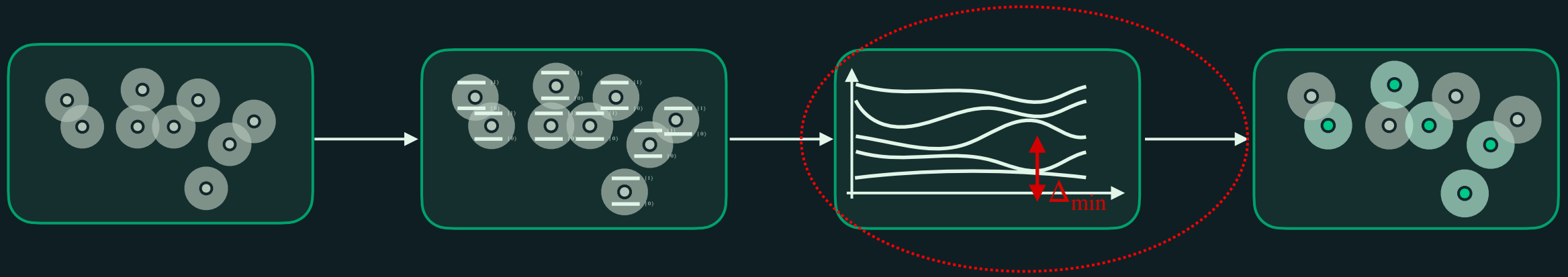
Quantum annealing



Success depends on $\Delta_{\min}$

For random instance of dancers, very hard to estimate $\Delta(s)$

Quantum annealing



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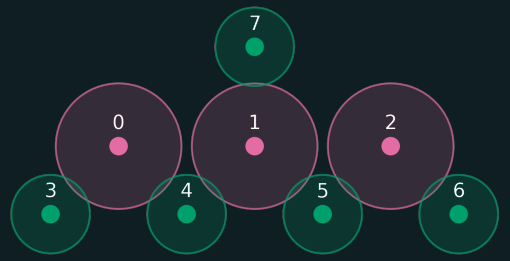
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Δ_{min} is small usually when MIS and near MIS configurations are separated by a barrier

Example of dancer configuration

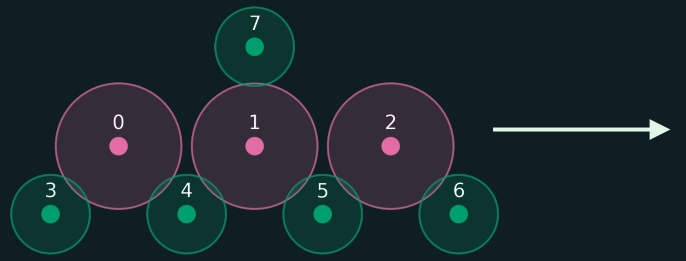
[DFJ02] M.Dyer, A.Frieze, M.Jerrum, "On counting independent sets in sparse graphs"

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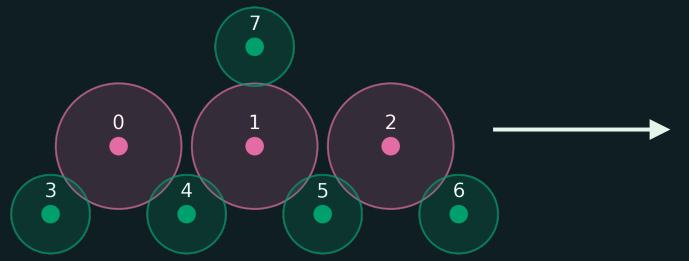
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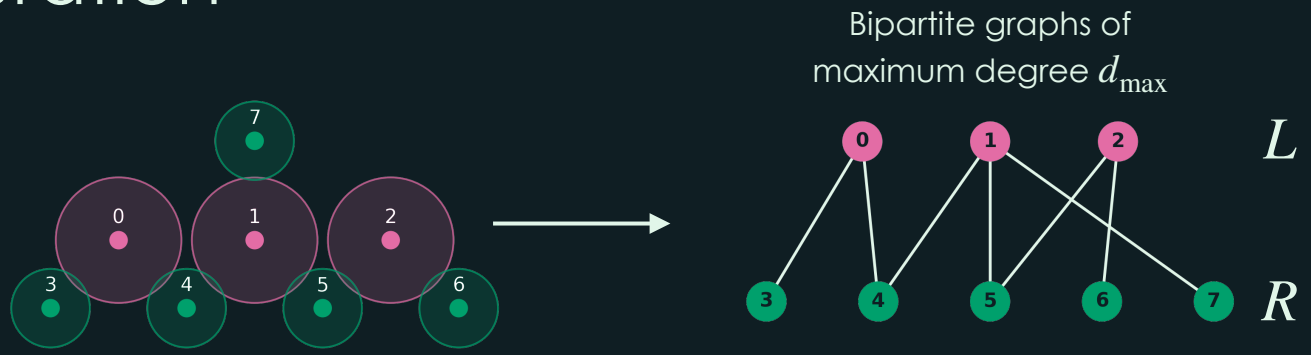
Example of dancer configuration

Bipartite graphs of
maximum degree $d_{\max}$



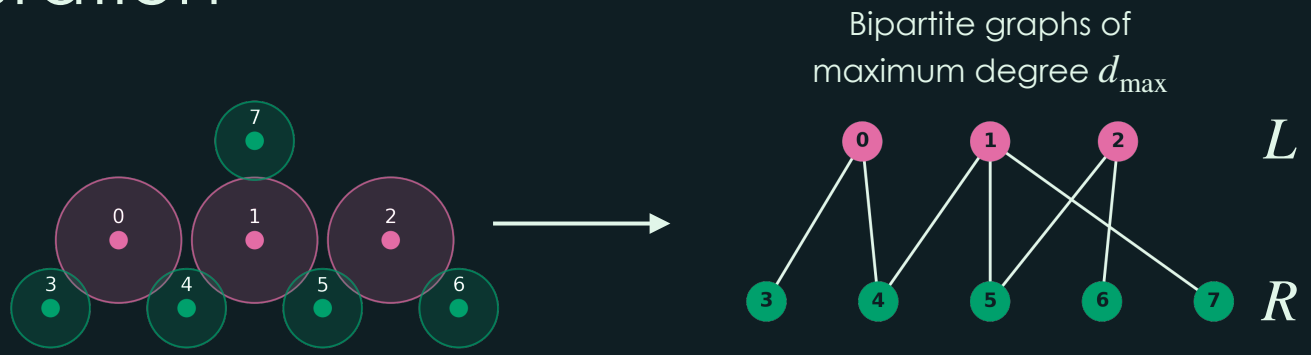
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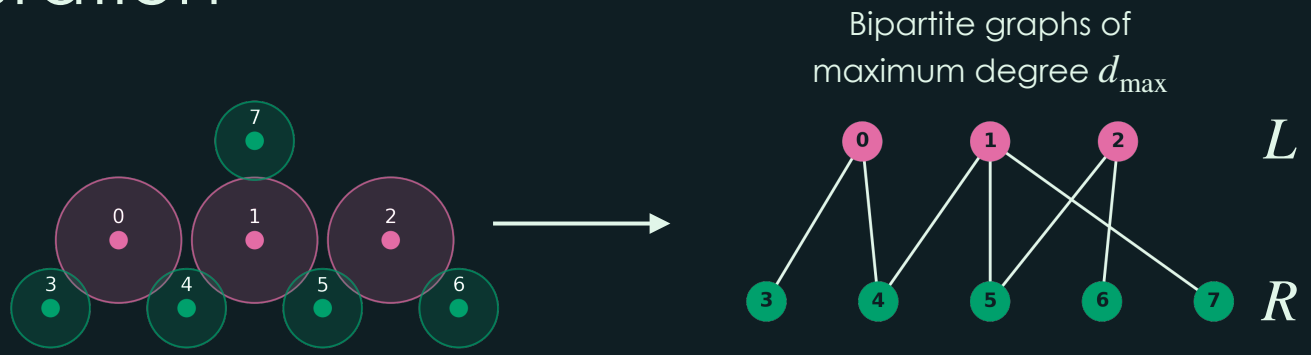
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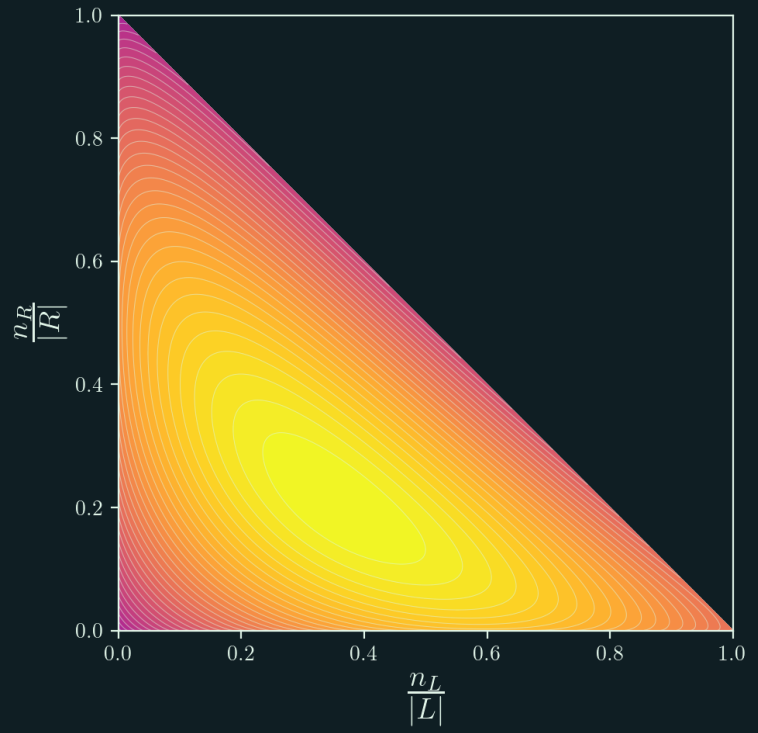
Average density of independent sets with n_L nodes in L, and n_R nodes in R

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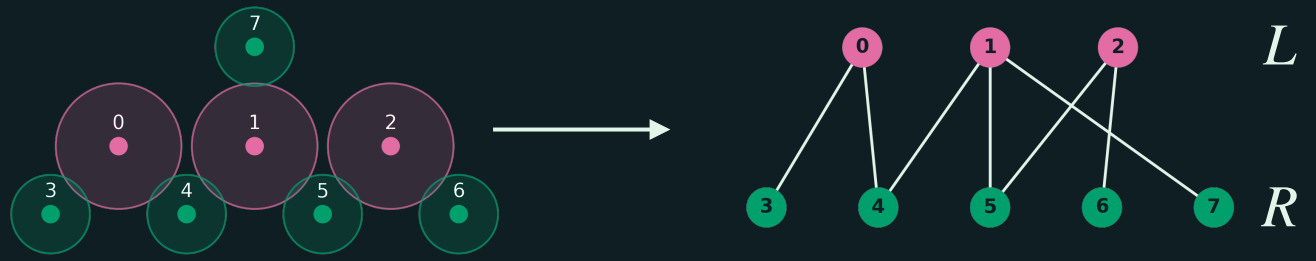


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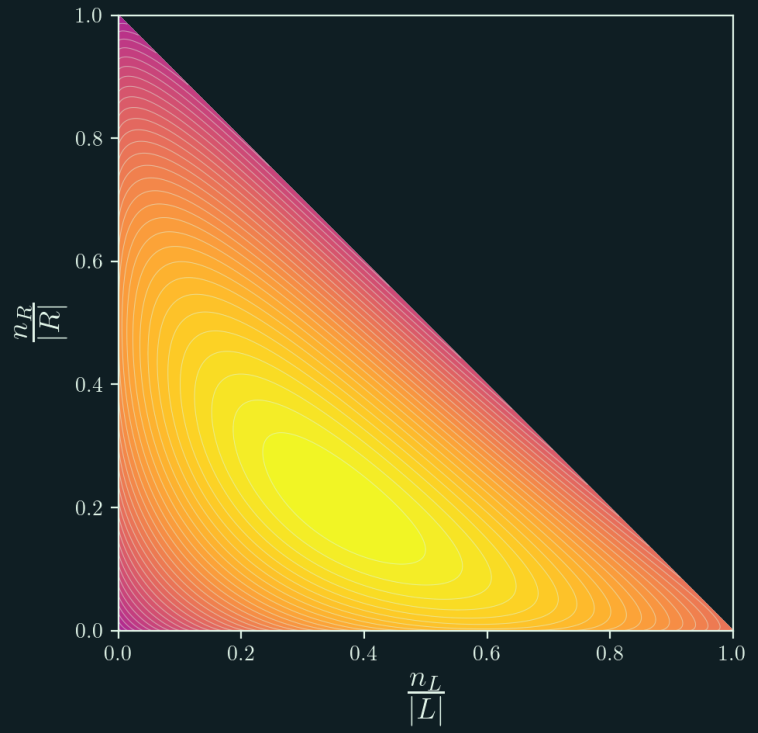


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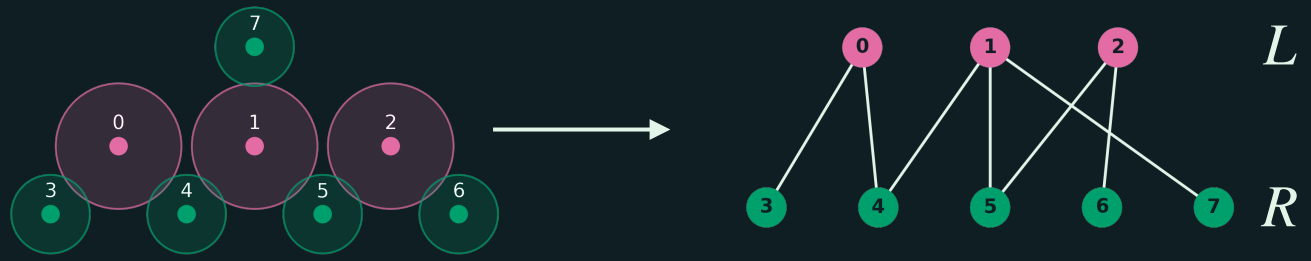
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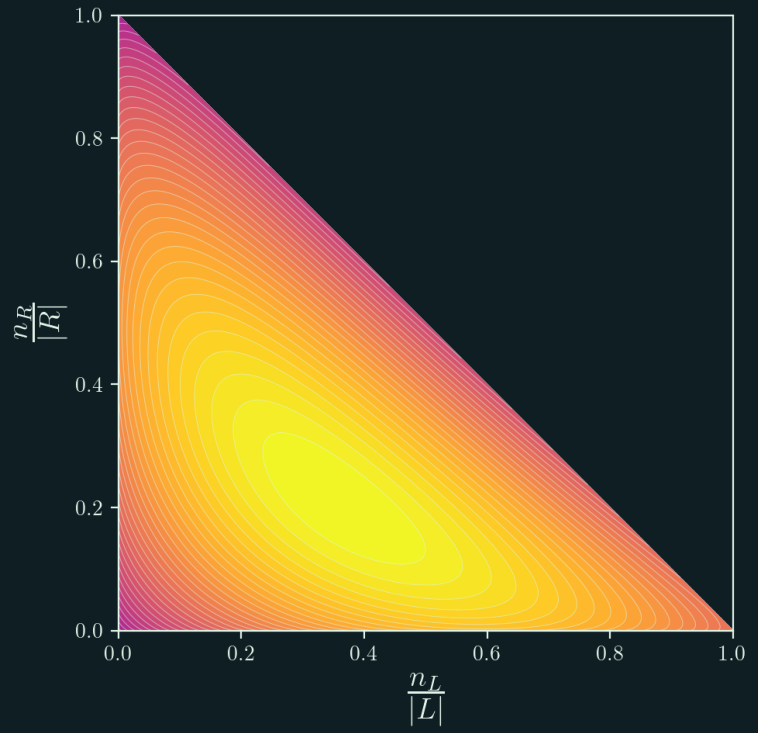
Increase $d_{\max}$

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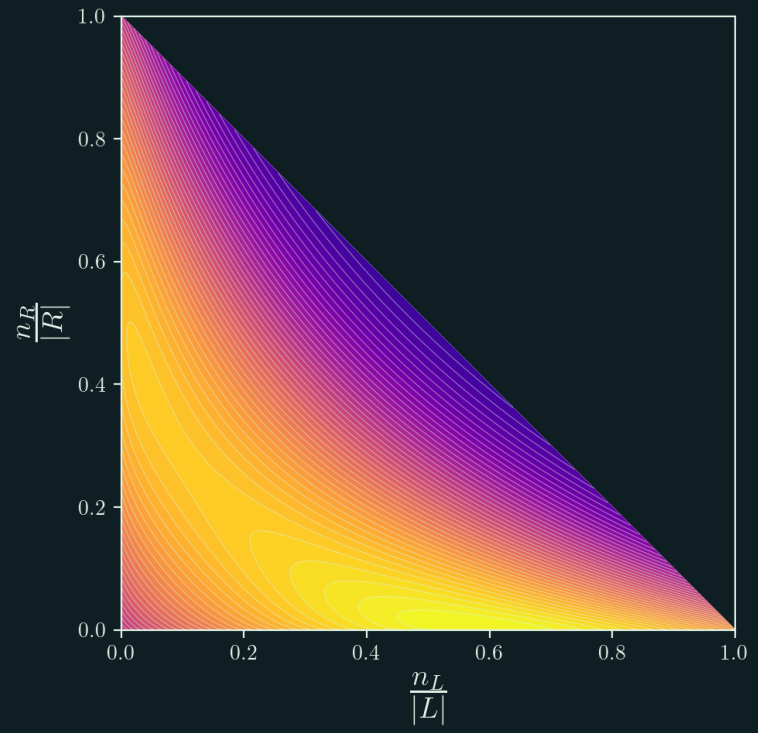
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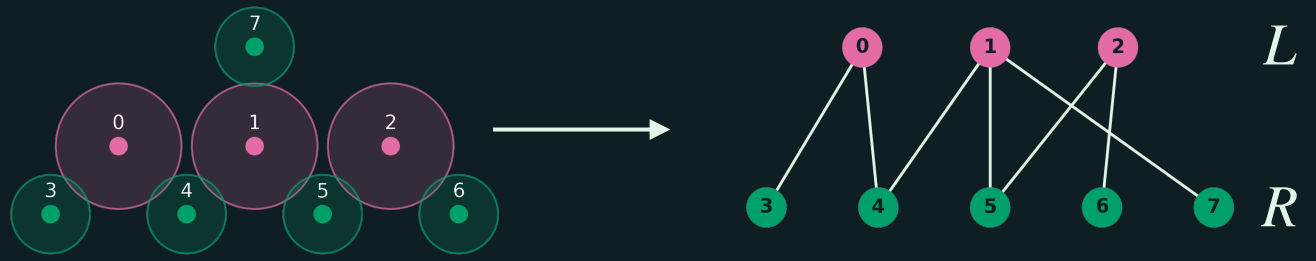


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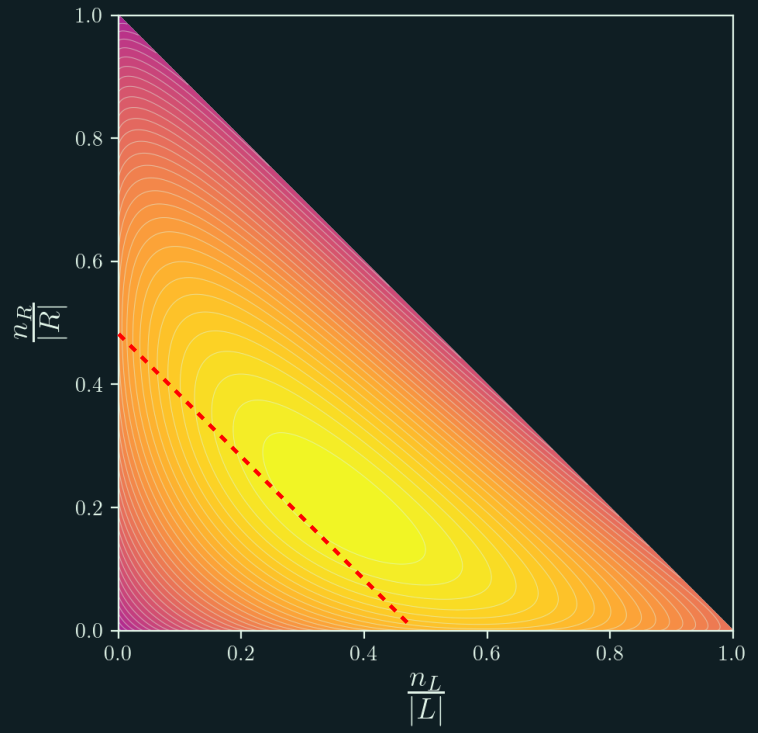


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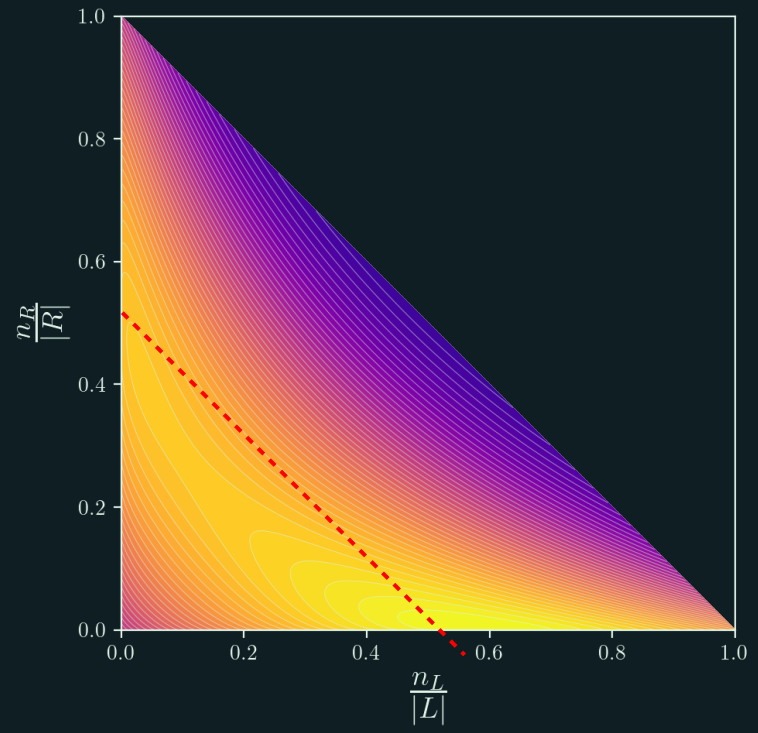
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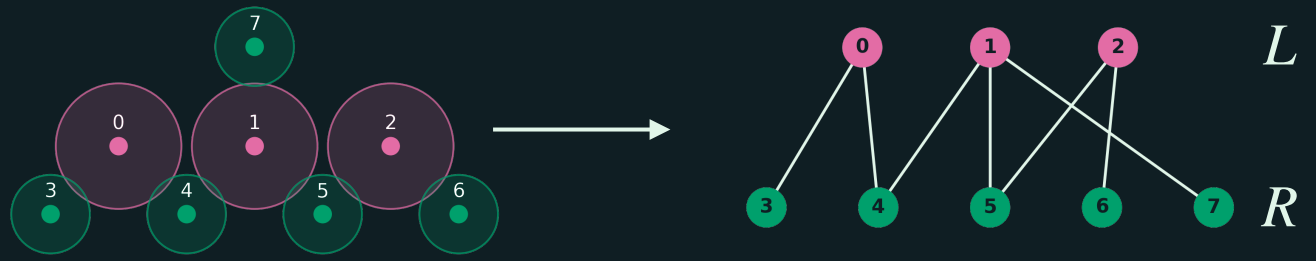


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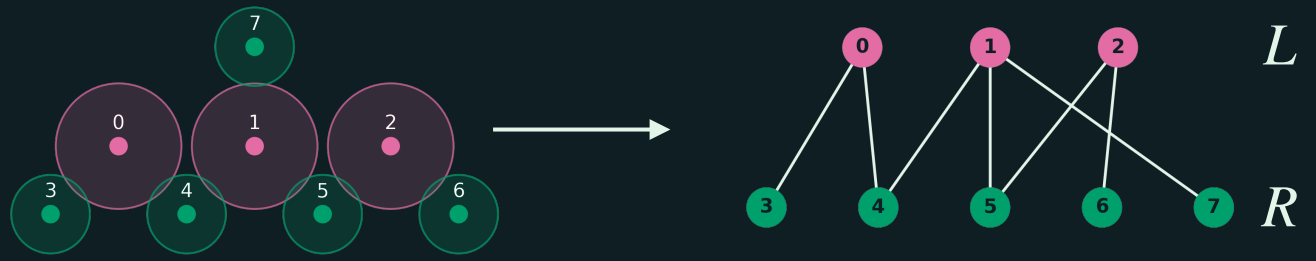
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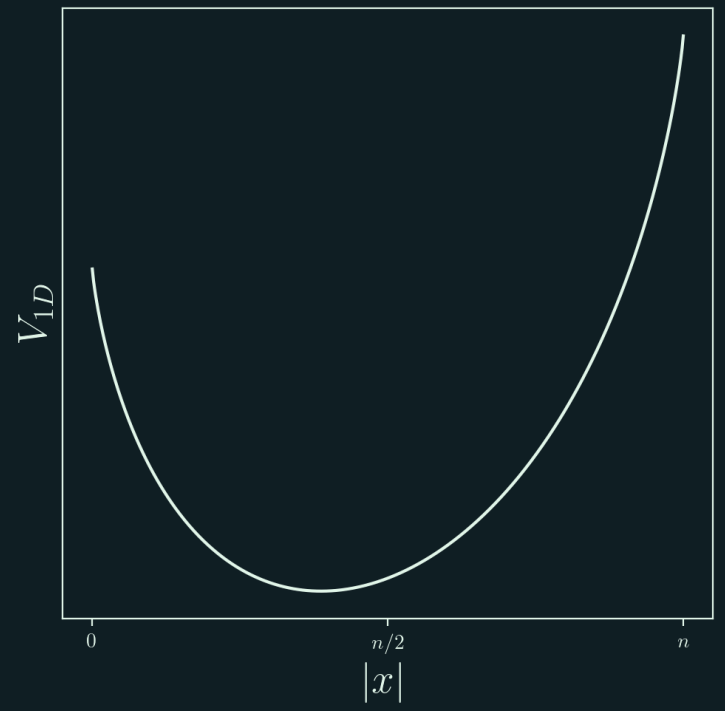


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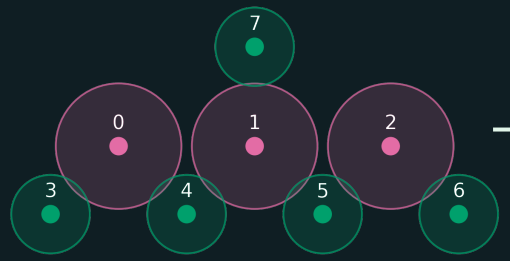
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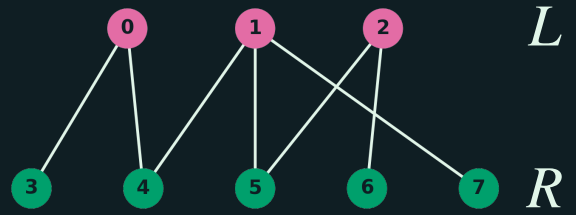
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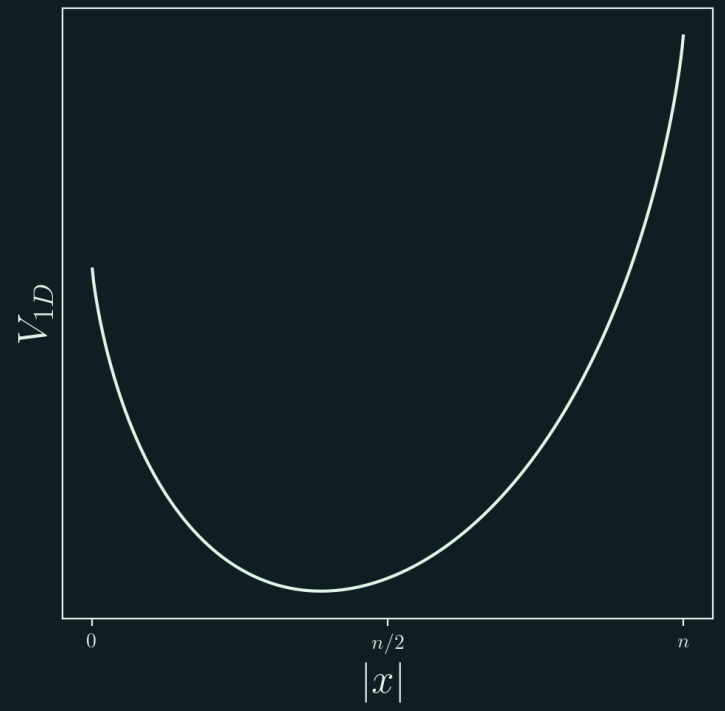
Example of dancer configuration



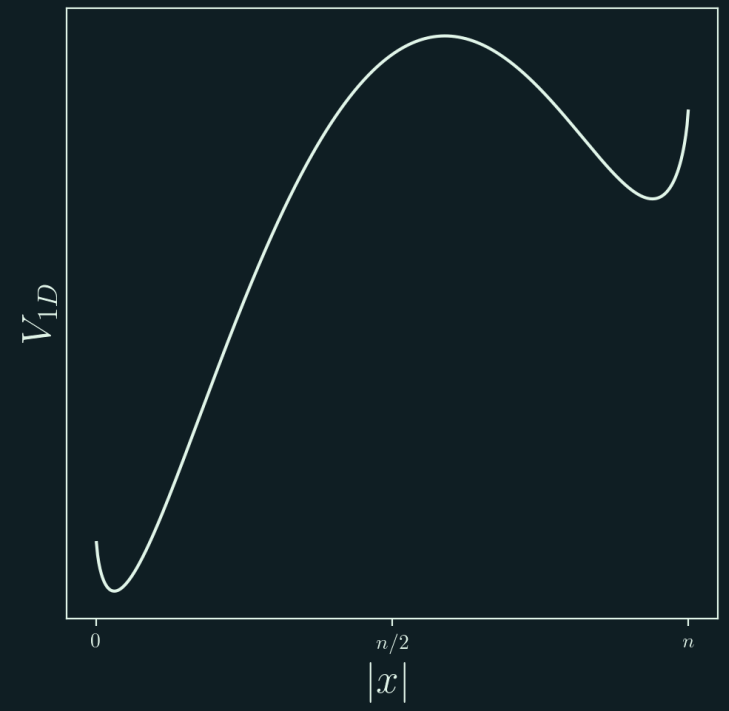
Bipartite graphs of maximum degree $d_{\max}$



Average density of independent sets with n_L nodes in L, and n_R nodes in R

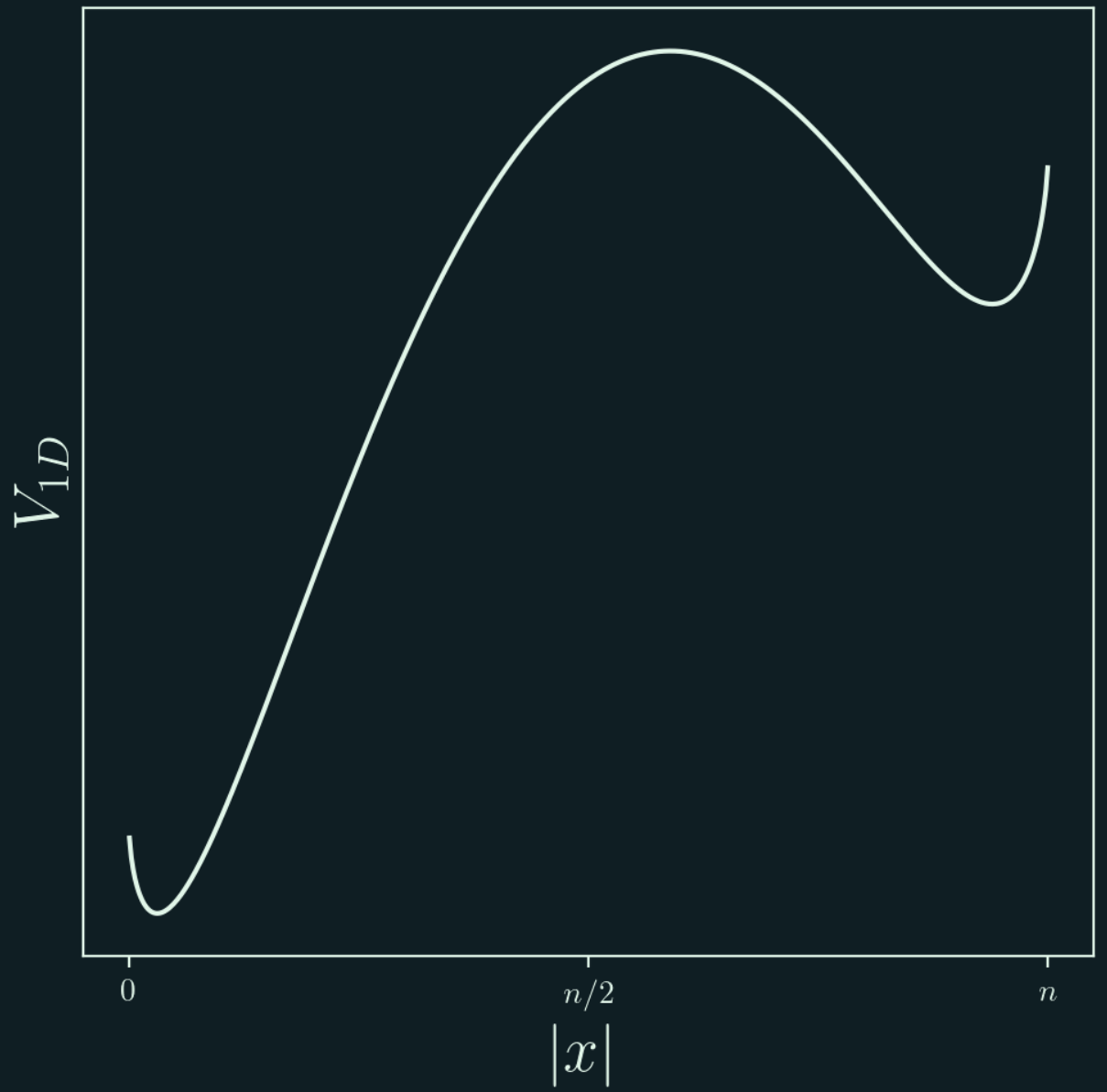


Increase $d_{\max}$

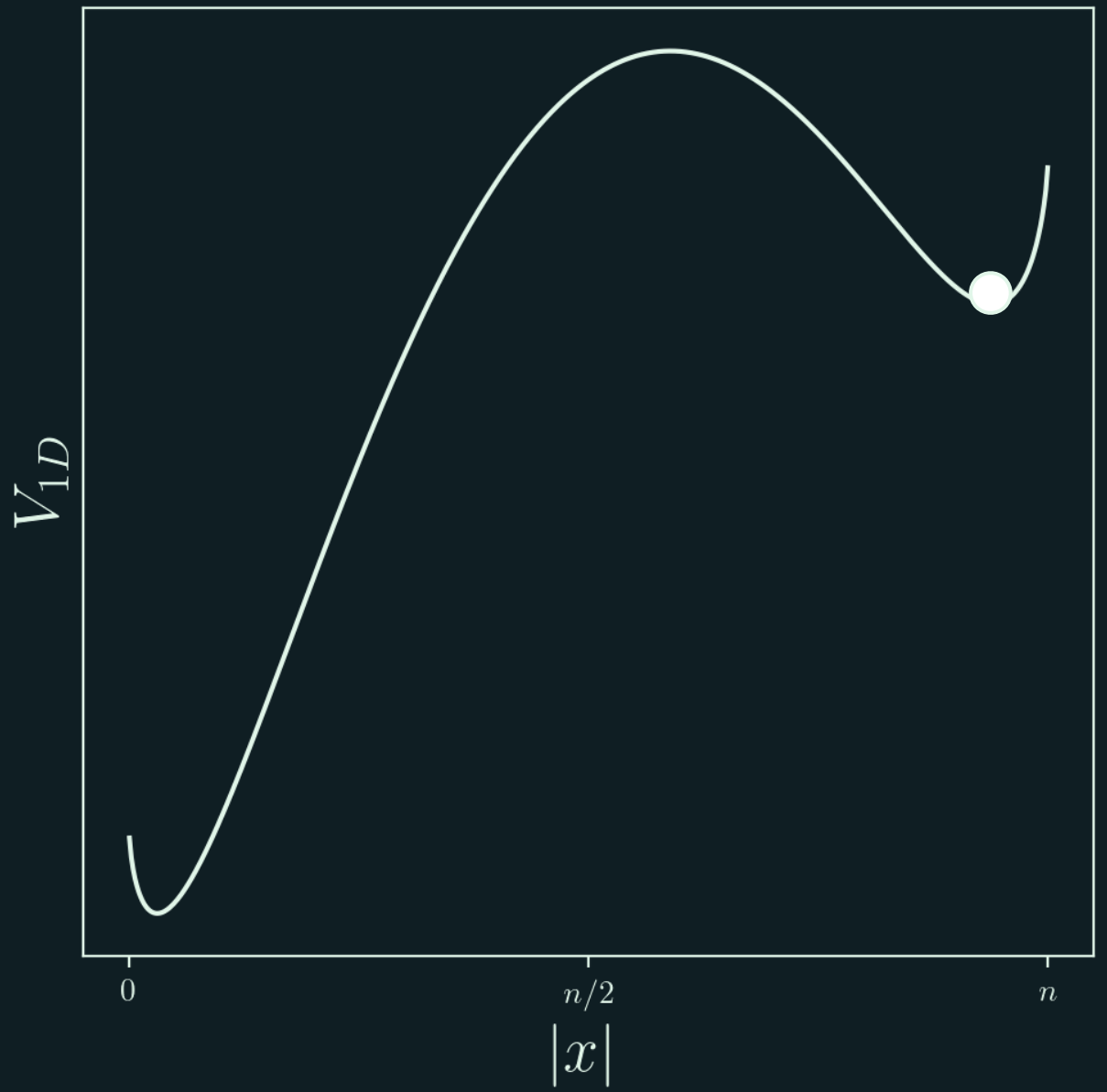


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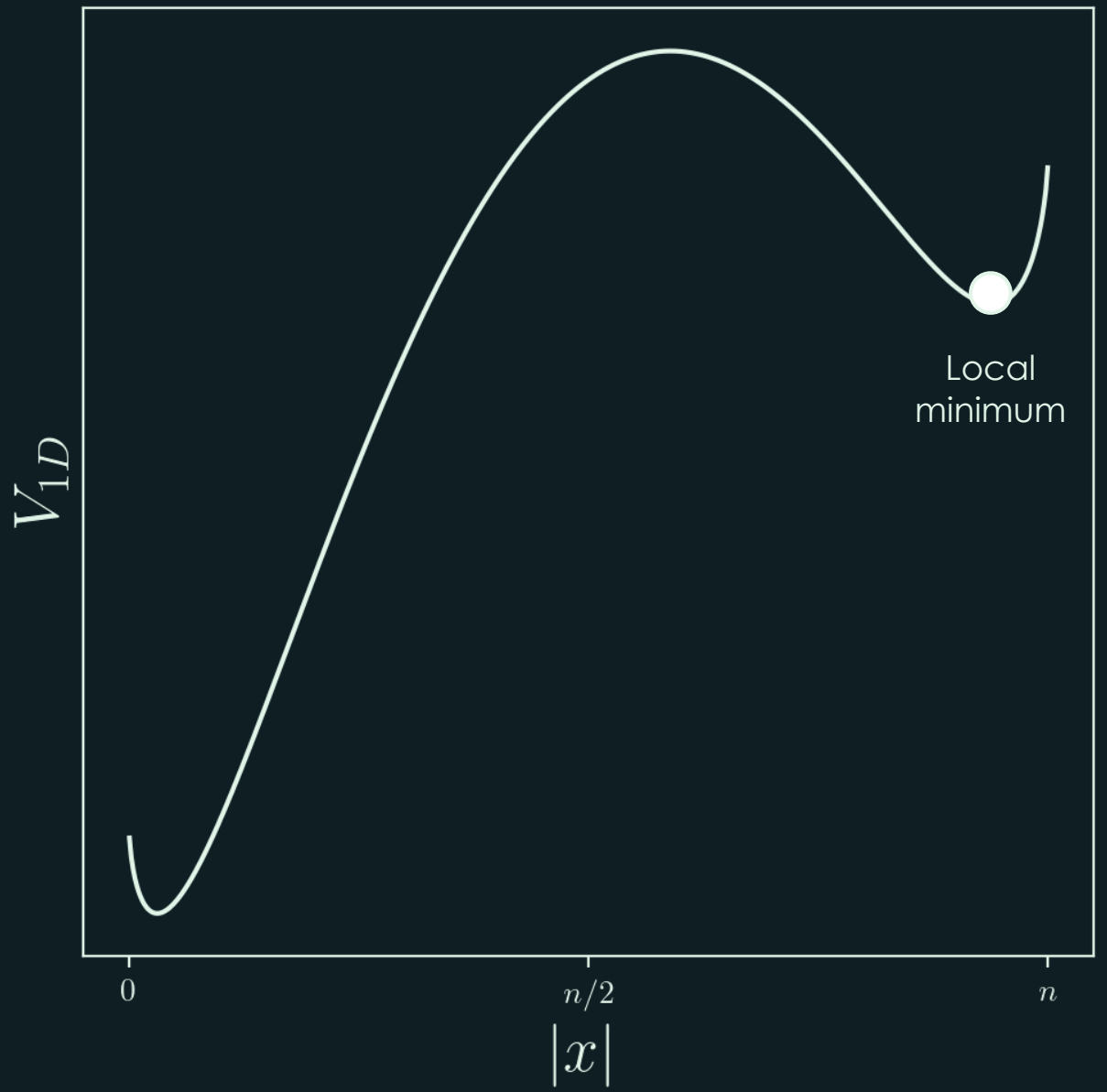
Sampling independent sets in bipartite graphs



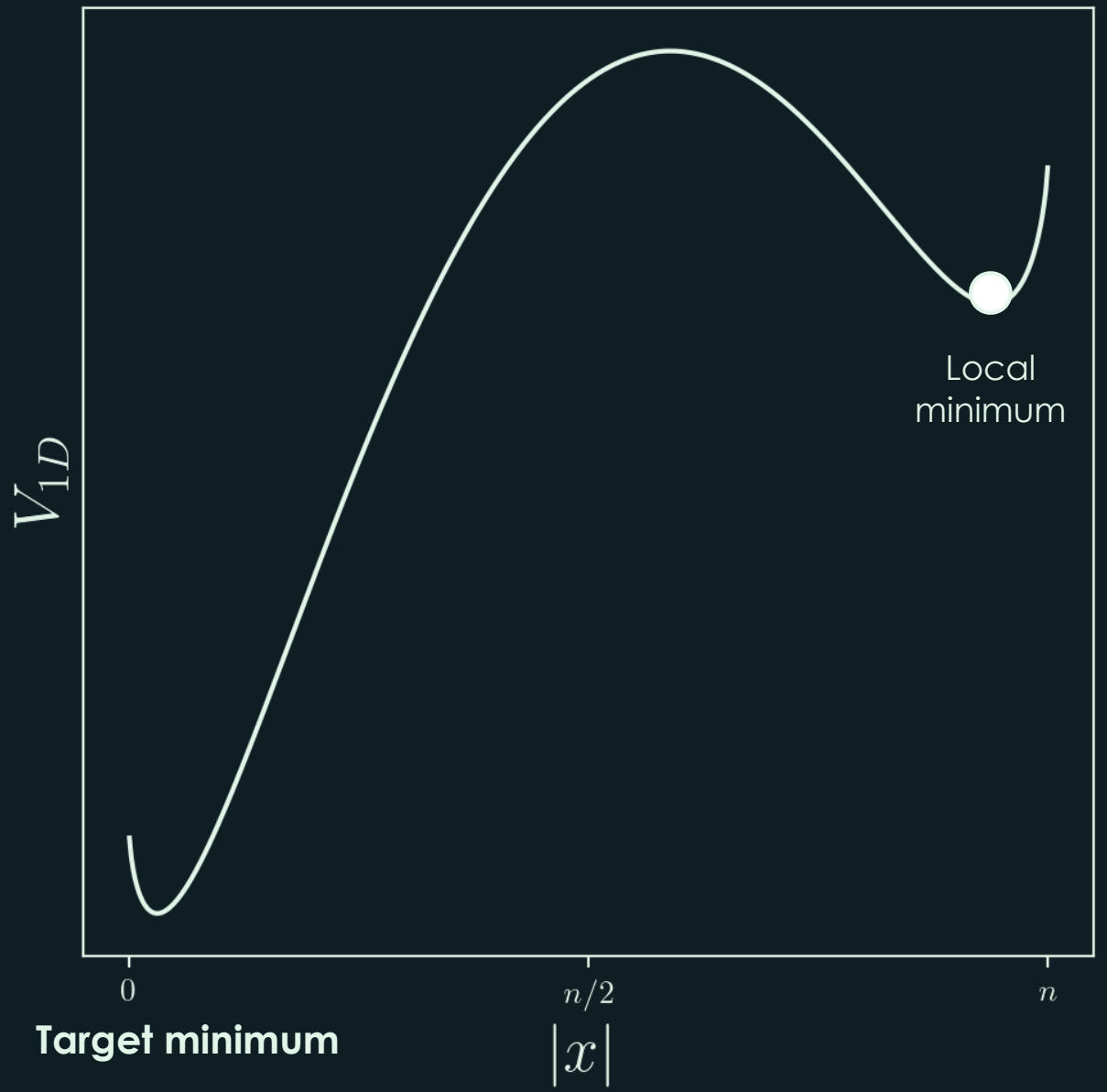
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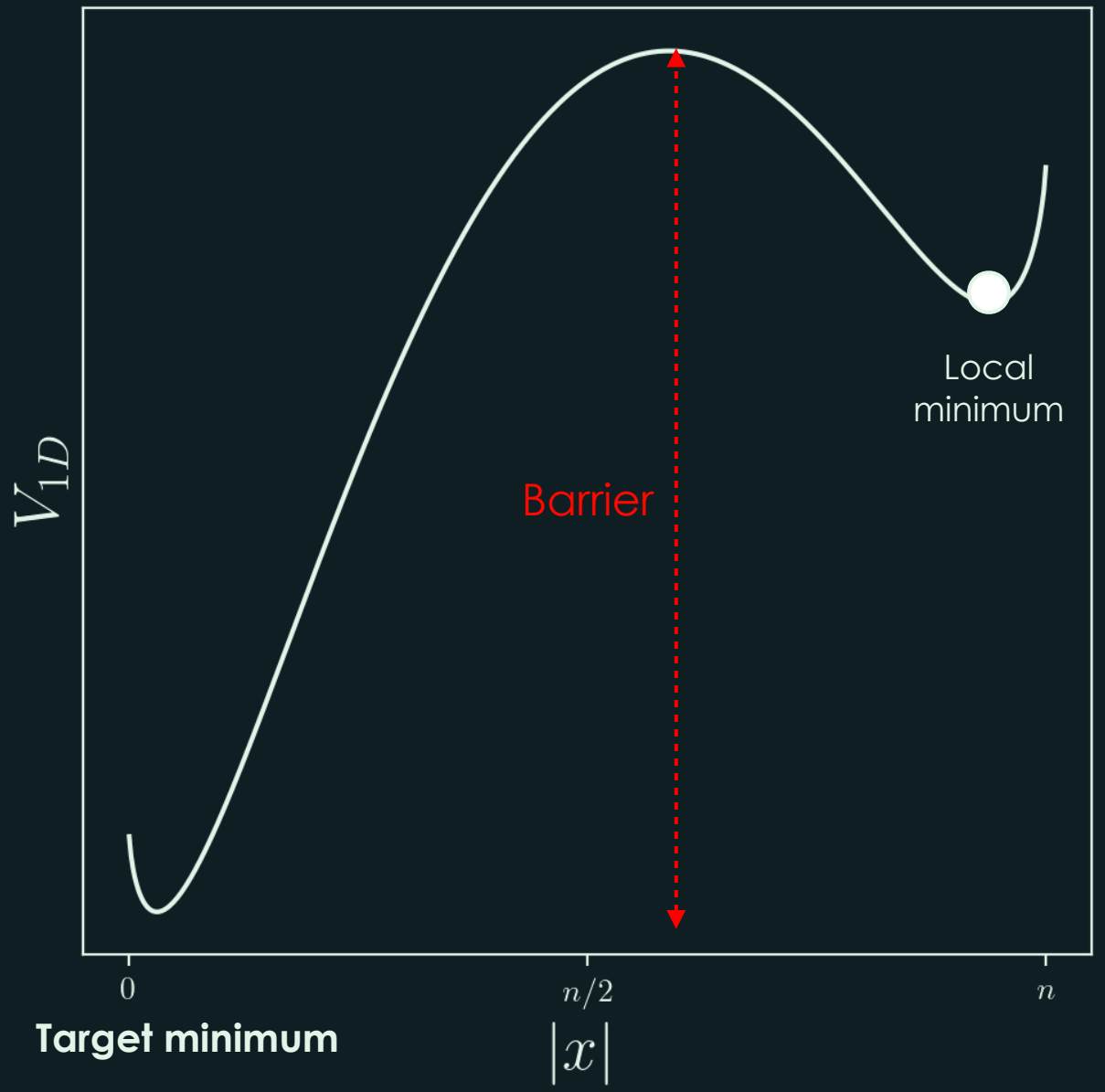
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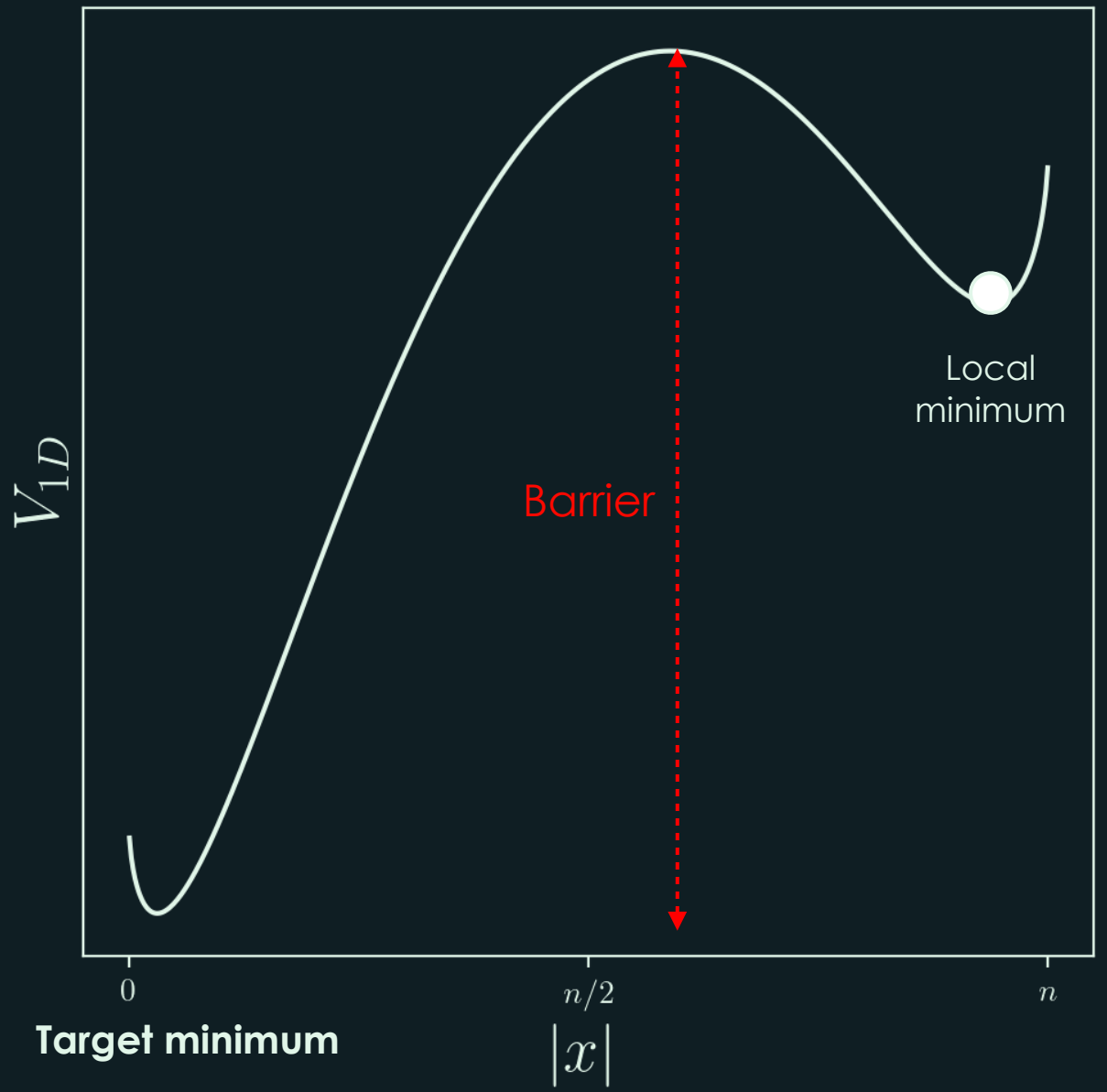
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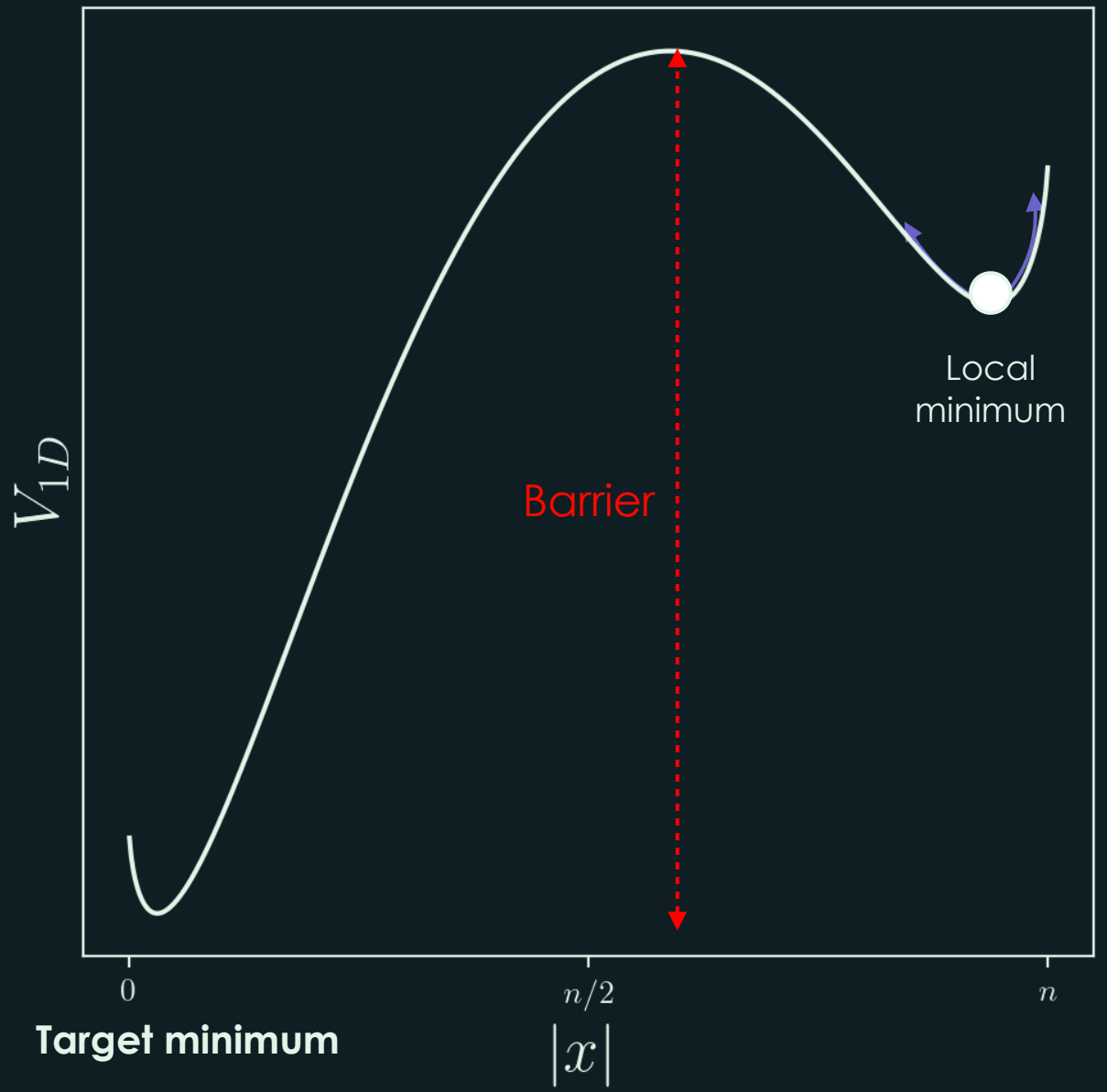


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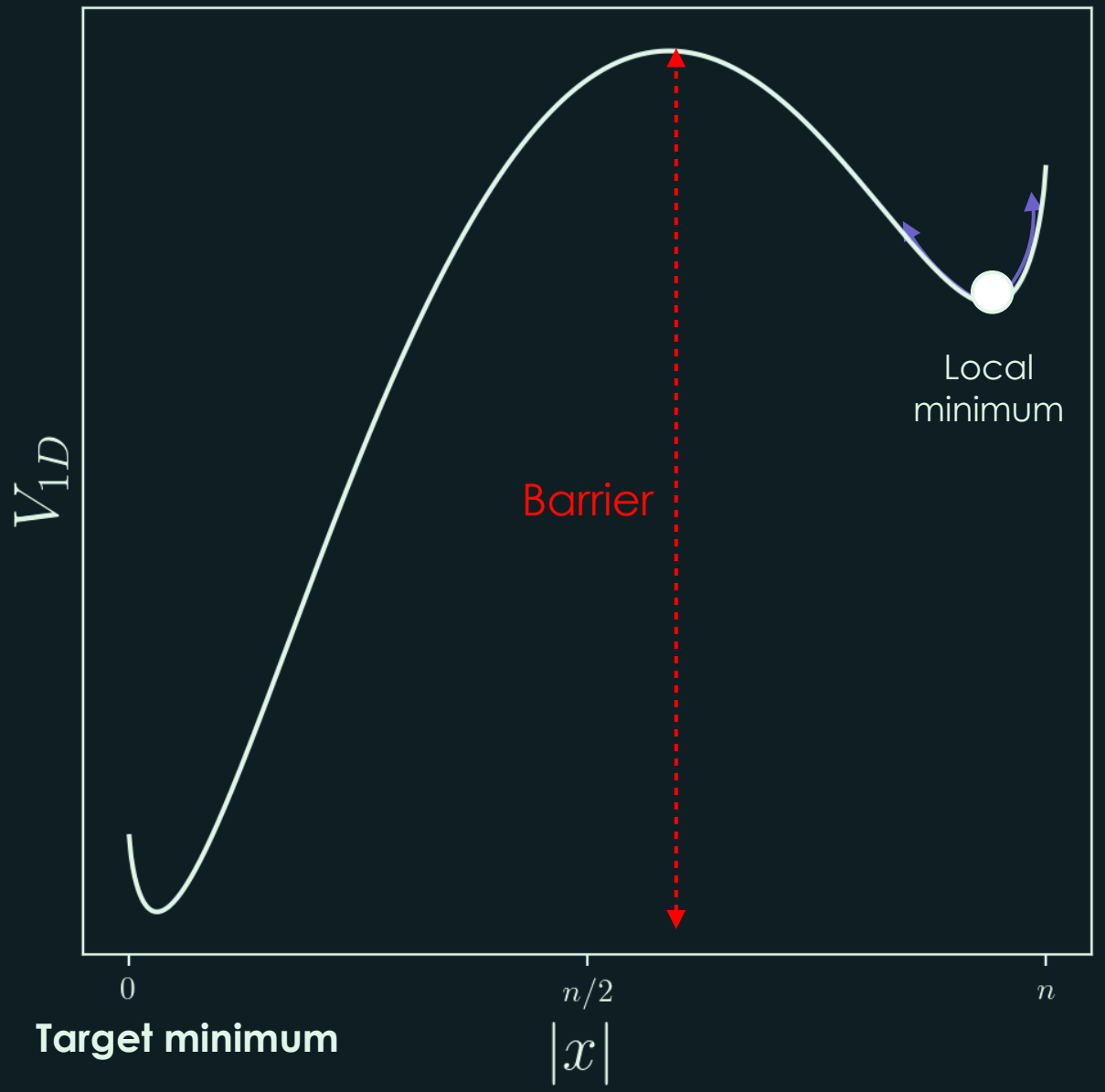
Classically

Sampling independent sets in bipartite graphs



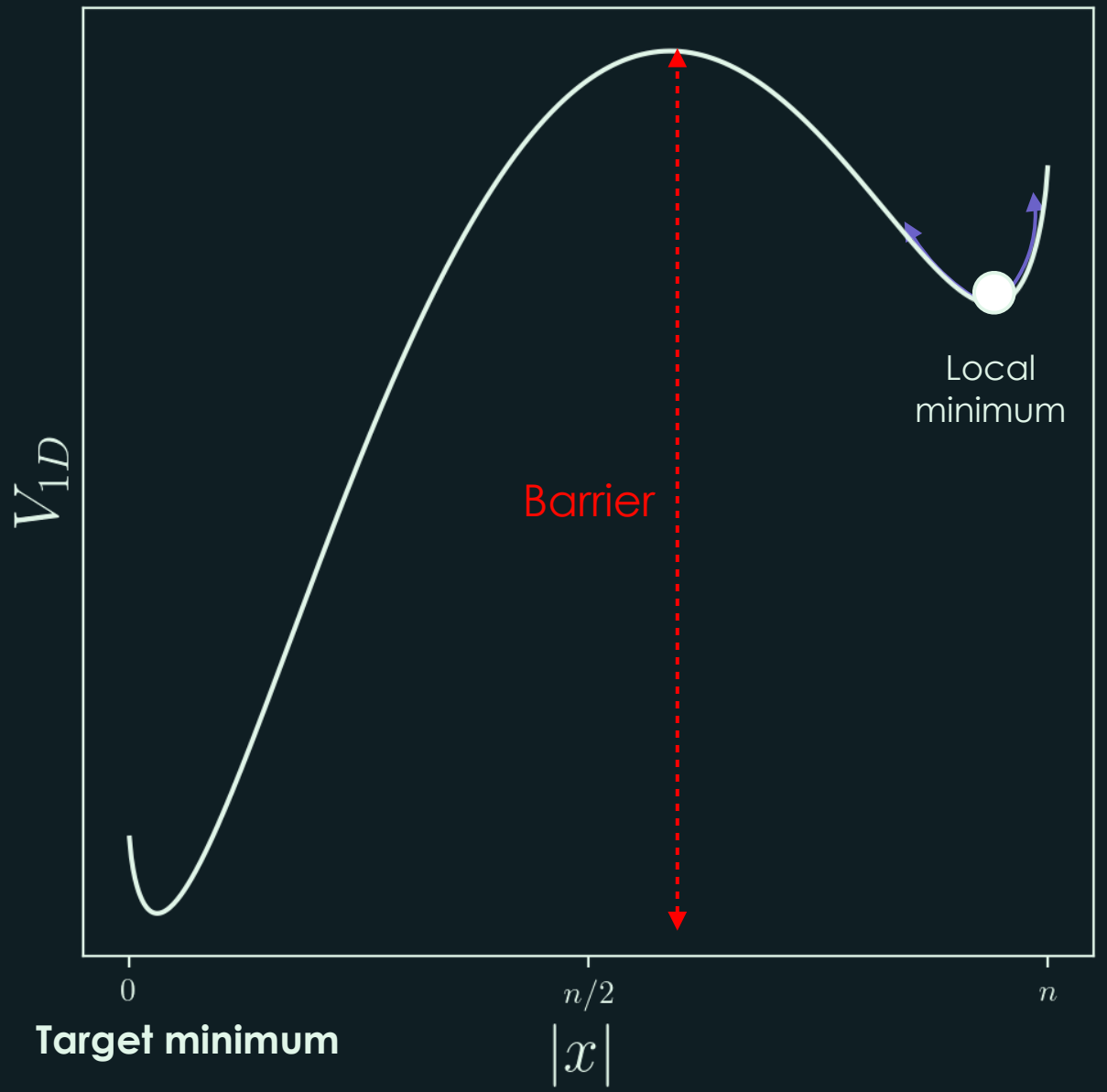
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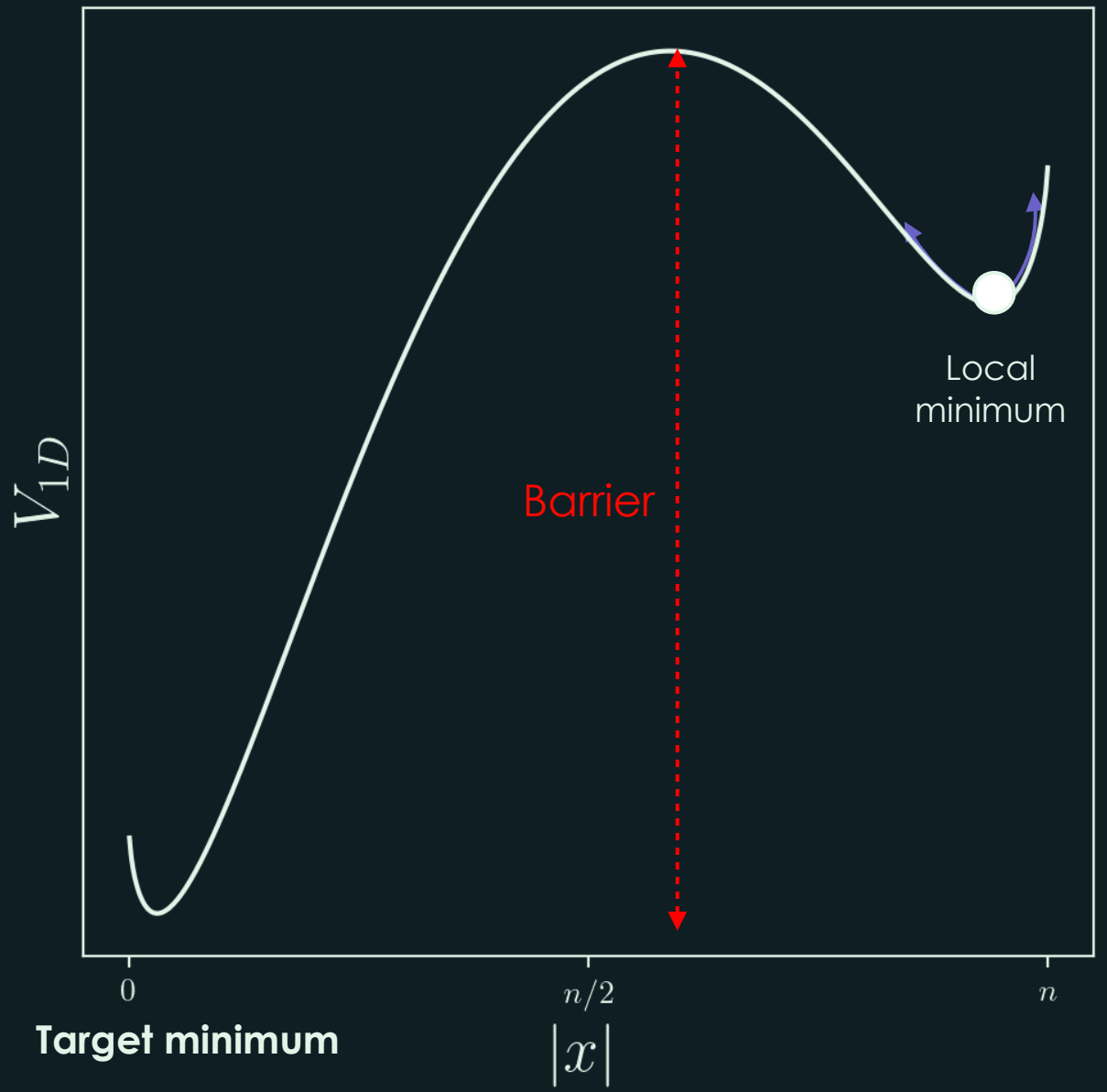
Sampling independent sets in bipartite graphs



Classically

For $d_{\max} \geq 6$, exponential time to escape local minima

Sampling independent sets in bipartite graphs

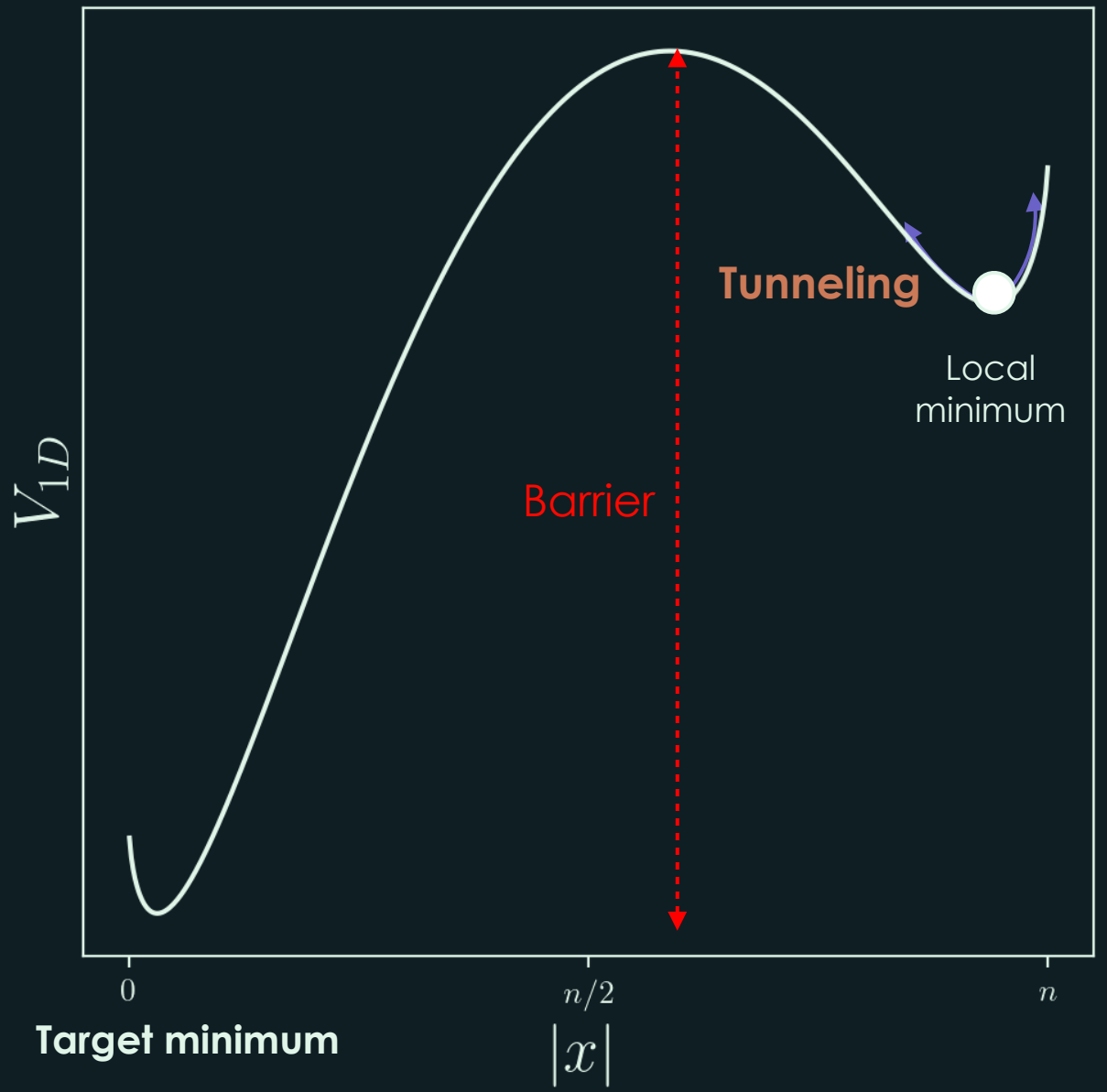


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Sampling independent sets in bipartite graphs

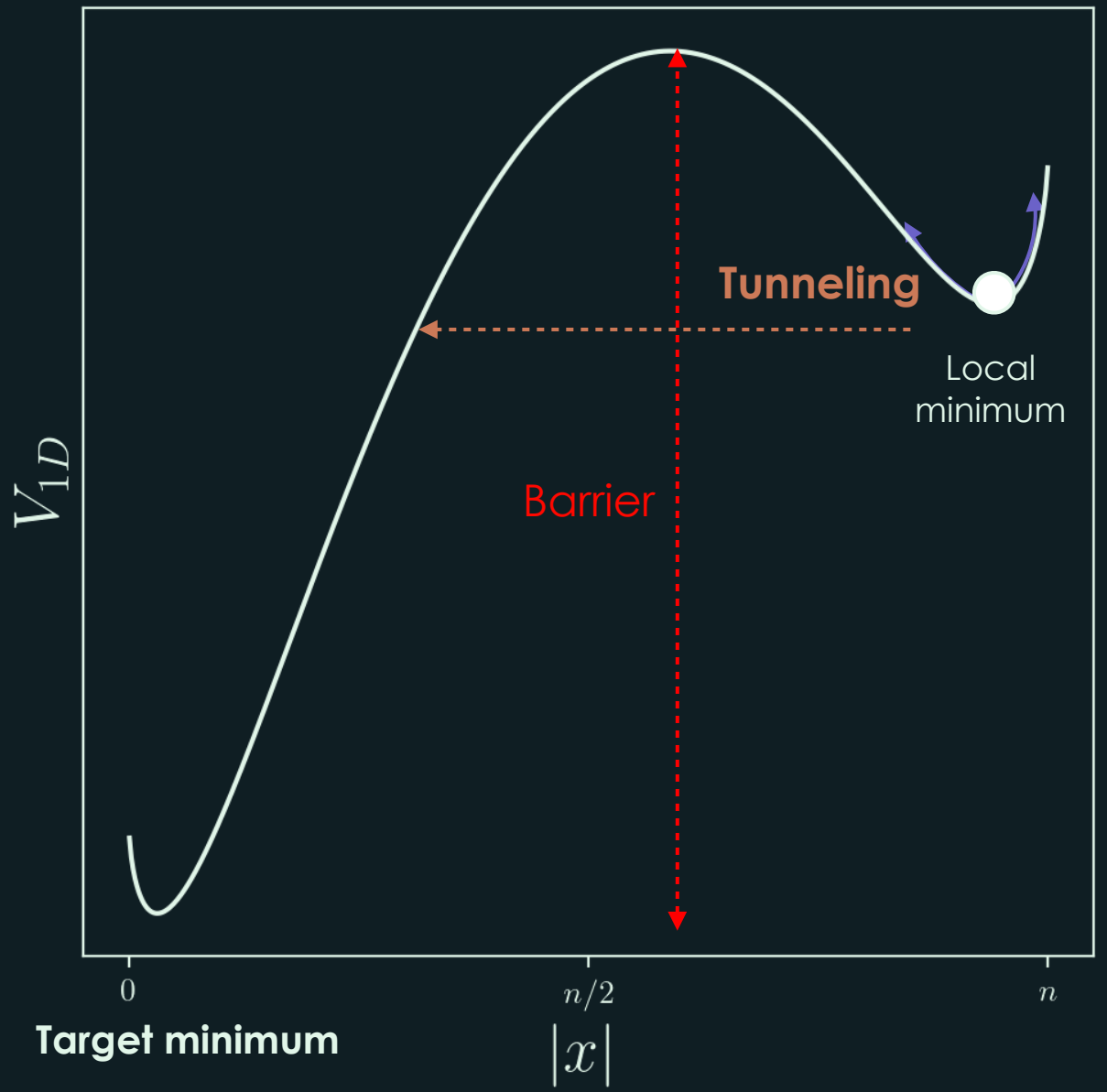


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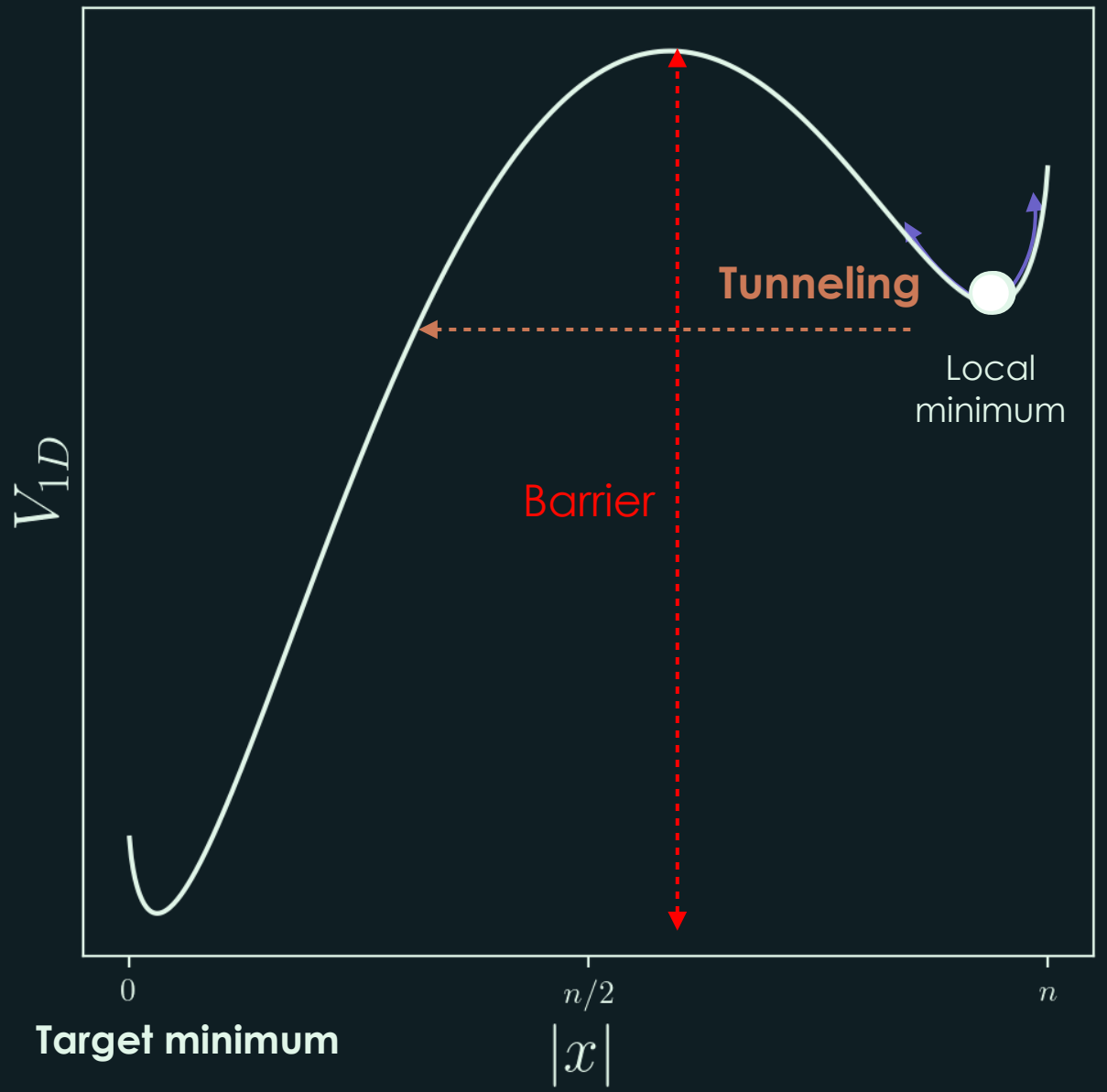


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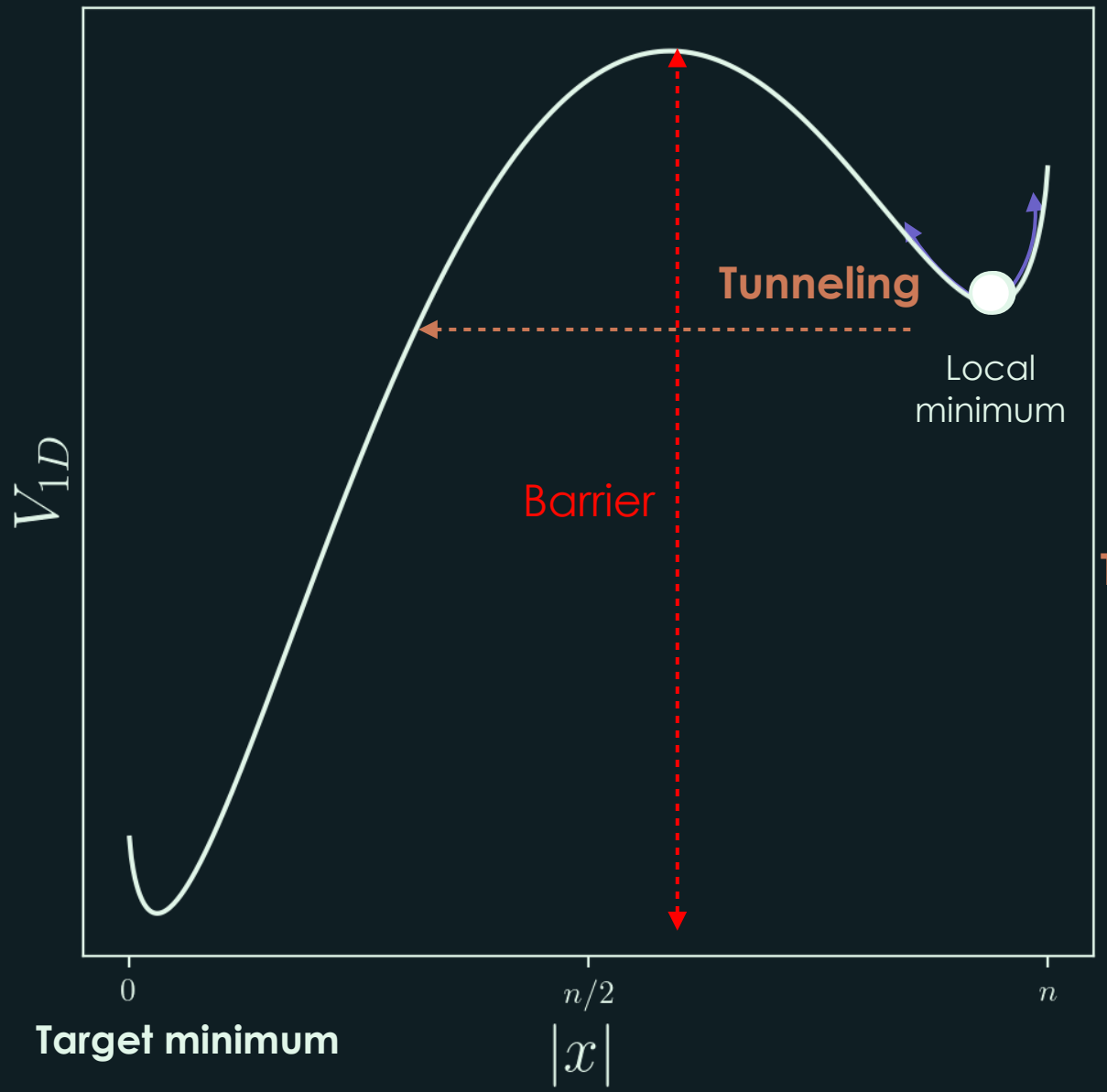


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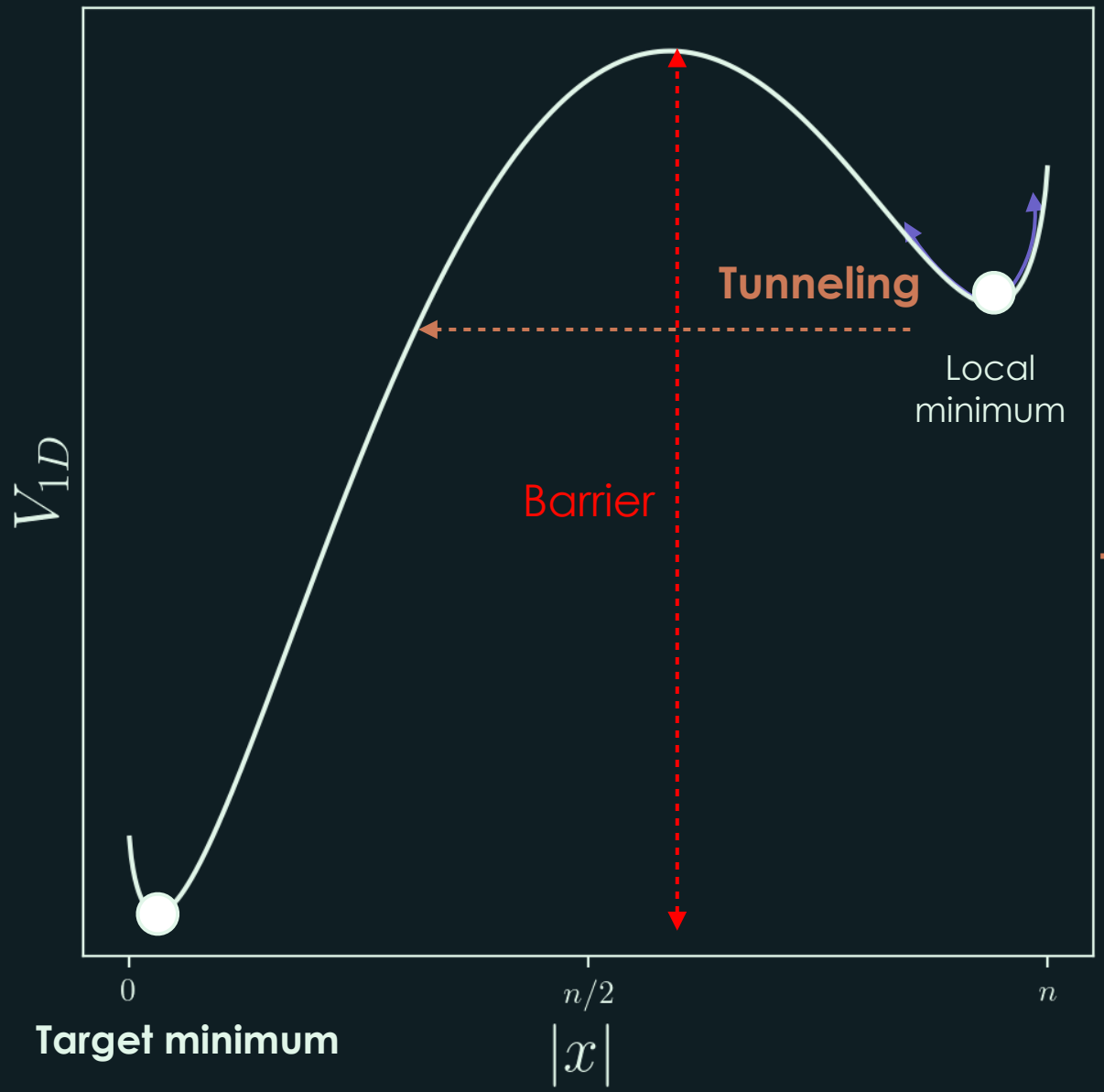
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Tunnel through the barrier → can we sample all of them?

Sampling independent sets in bipartite graphs



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Hamming weight with a spike - One known example

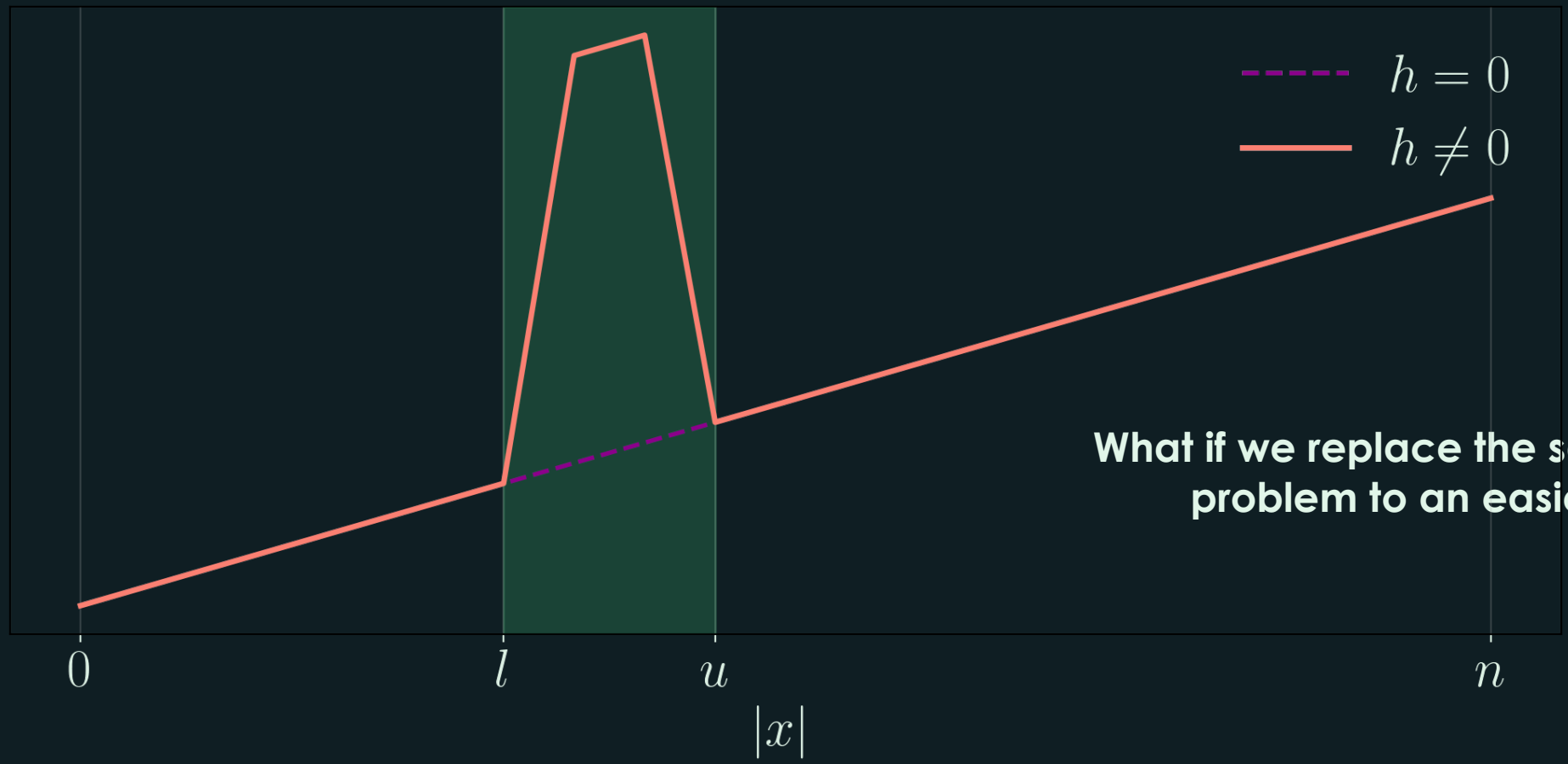
[Rei04] Ben W Reichardt. The quantum adiabatic optimization algorithm and local minima. In Proceedings of the thirty-sixth annual ACM symposium on Theory of computing, pages 502–510, 2004

Hamming weight with a spike - One known example

What if we replace the sampling MIS problem to an easier one?

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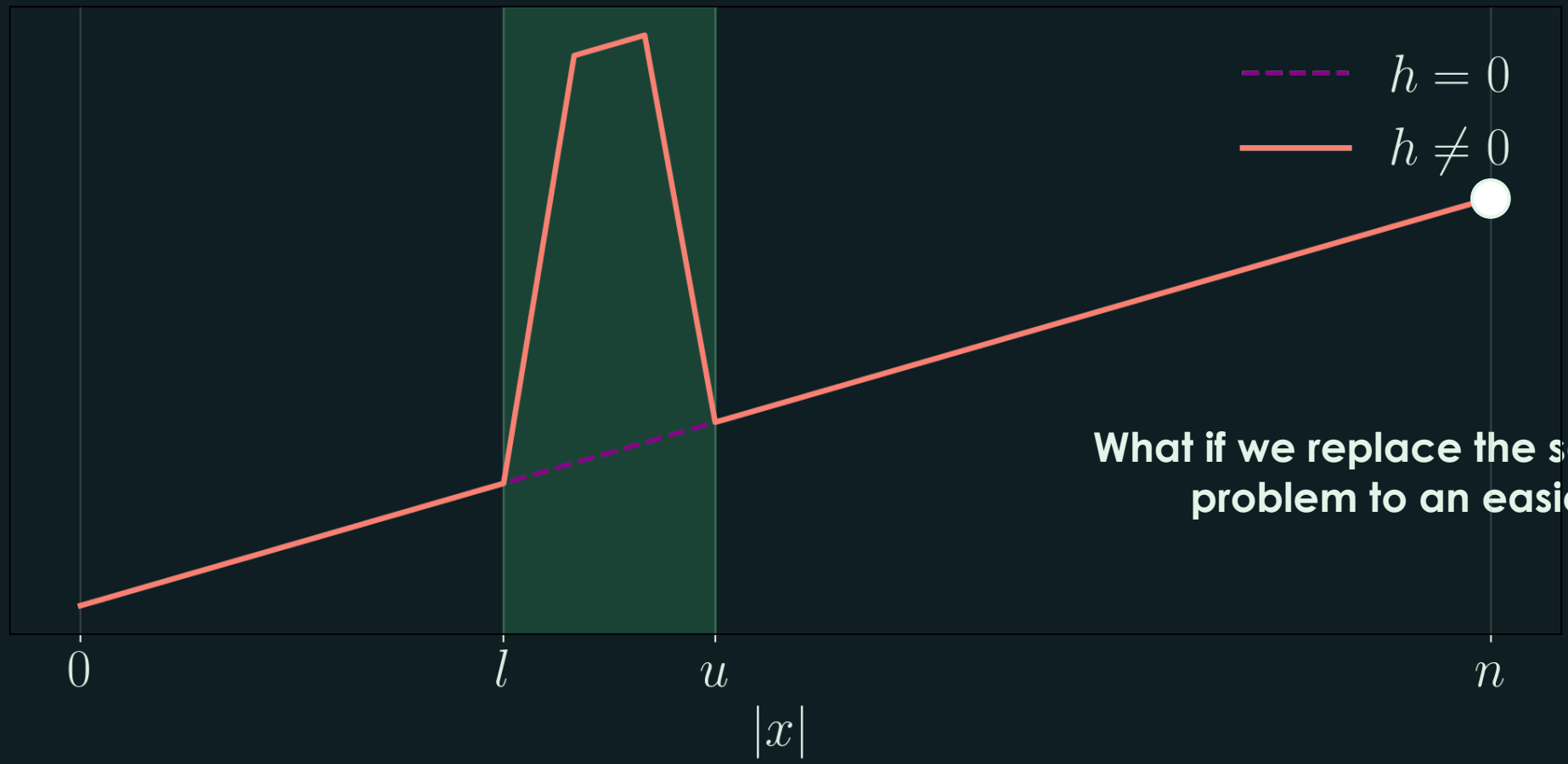
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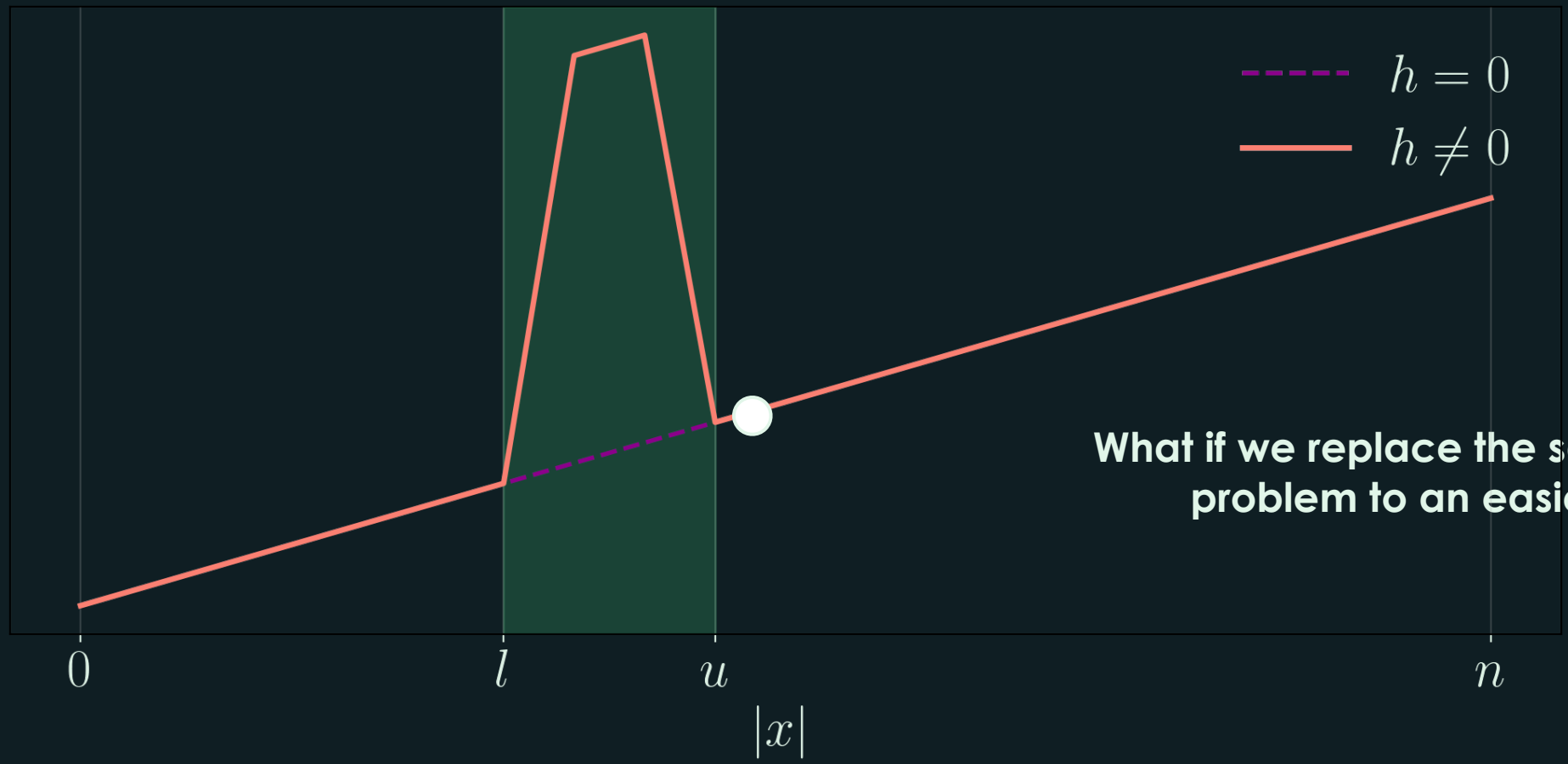
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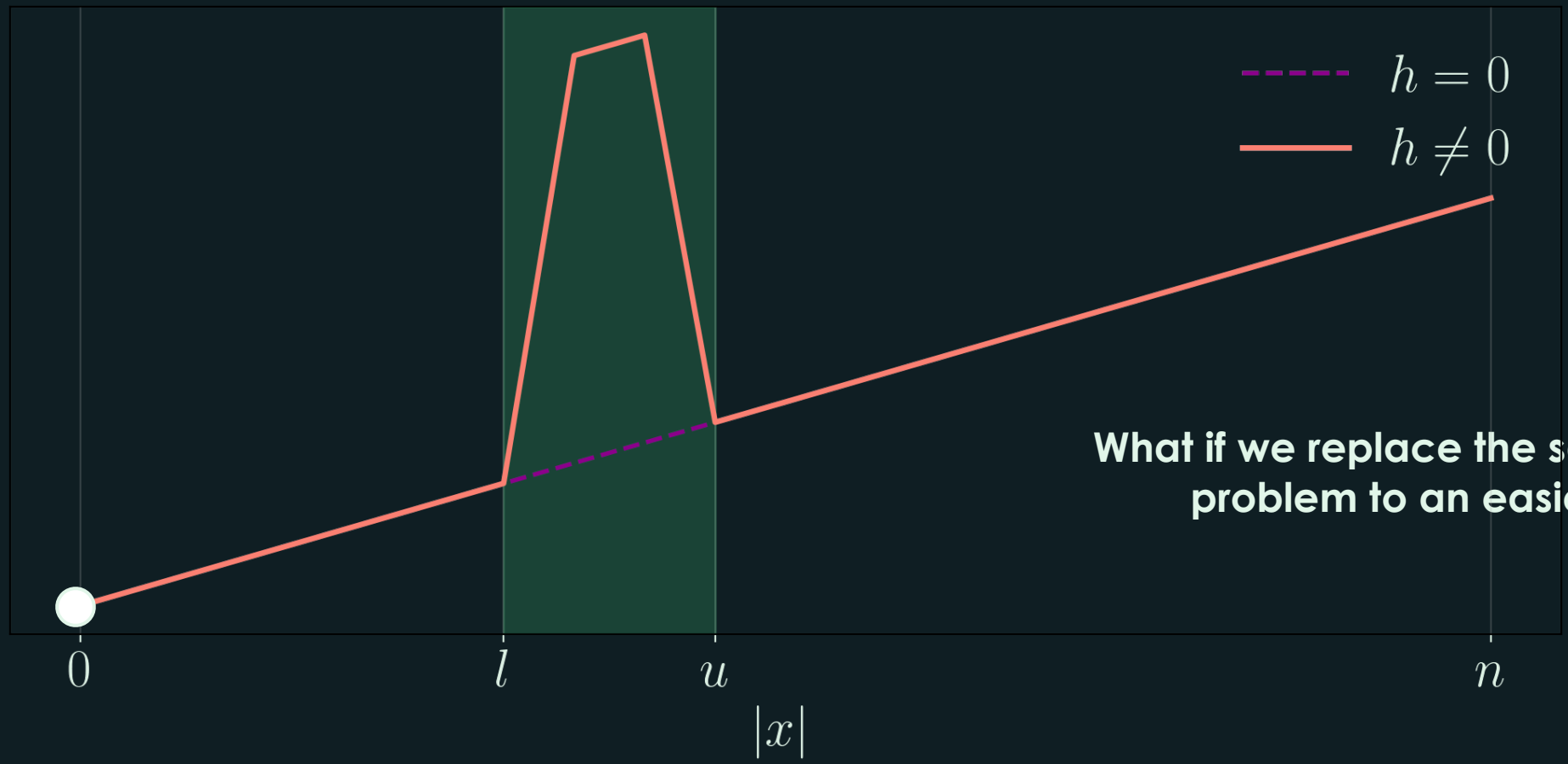
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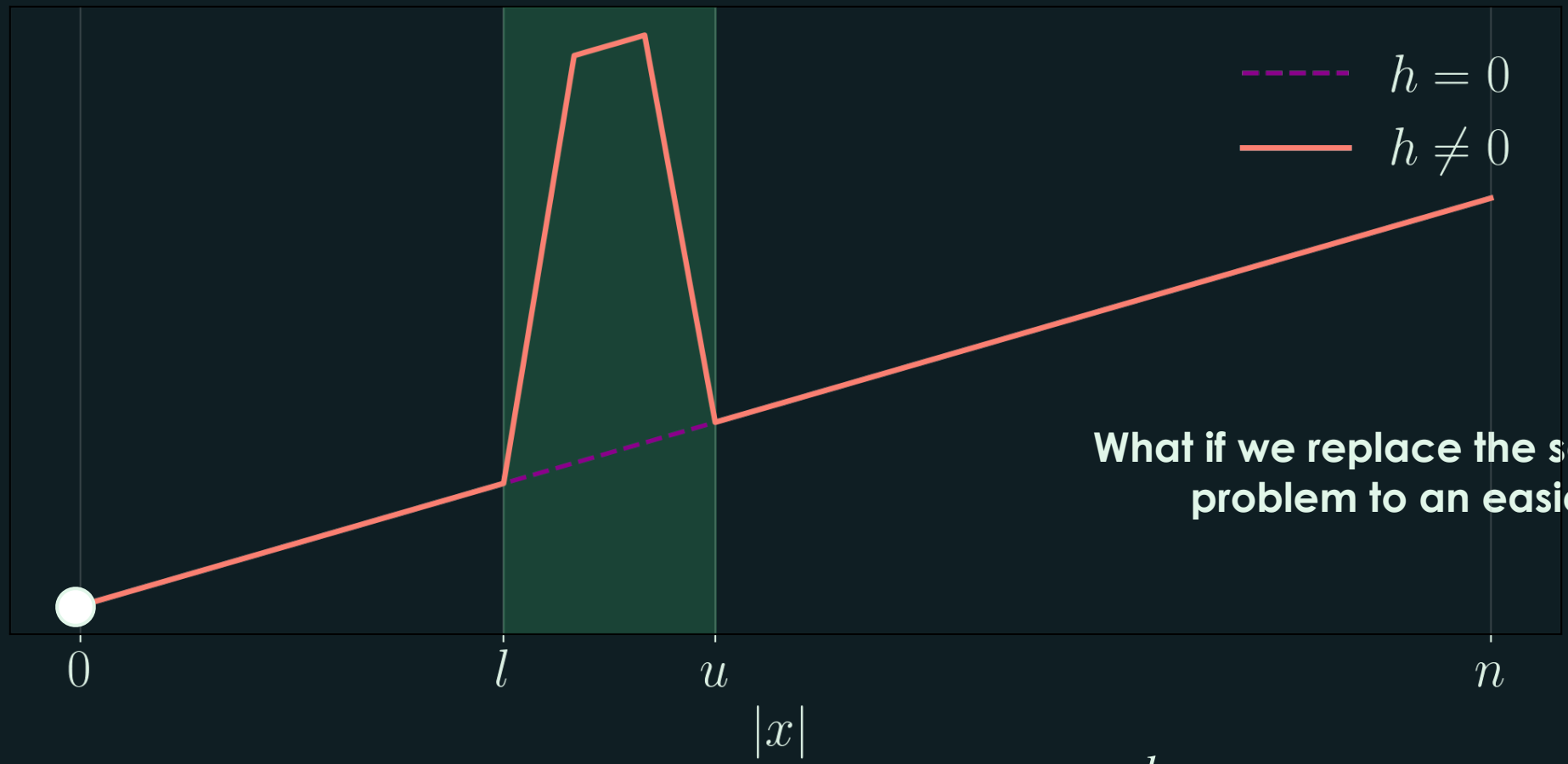
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What if we replace the sampling MIS problem to an easier one?

Ground state can be prepared in $O(\text{poly}(n))$ time if $h\frac{u-l}{\sqrt{l}} = O(1)$

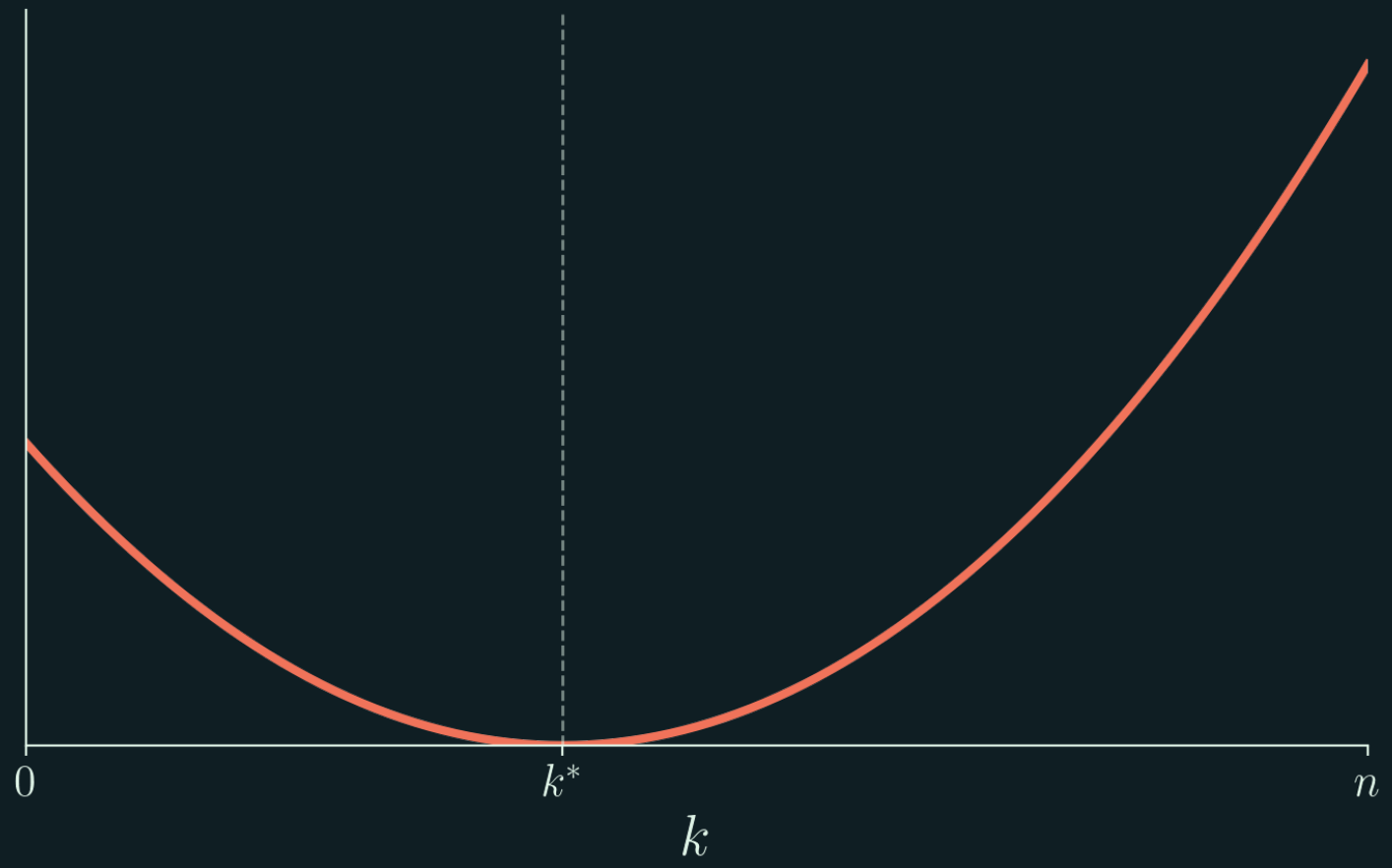
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Motivation

Goal: Minimizing V and the ground state should be peaked near the minimizer of V, and falling off fast away from it

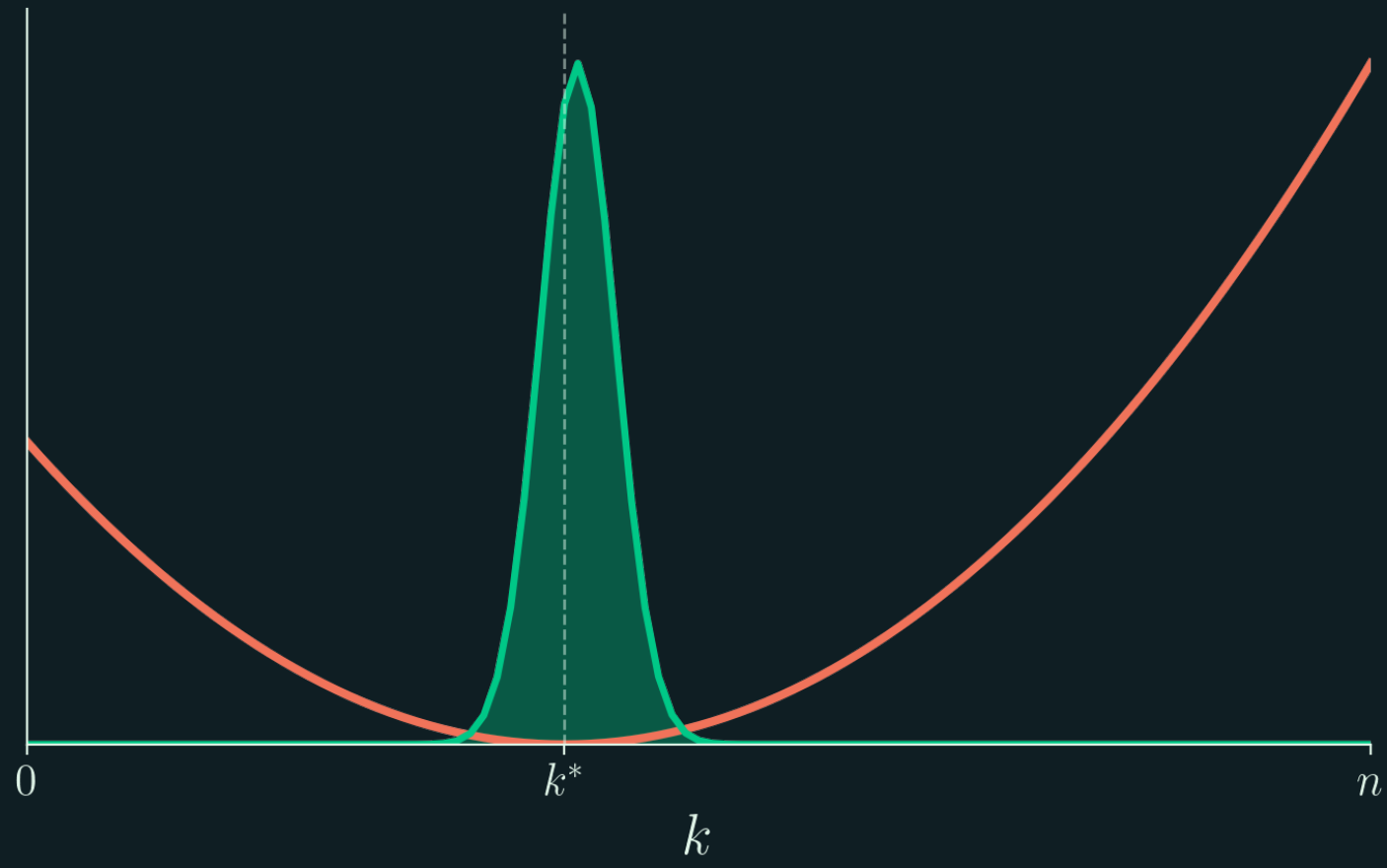
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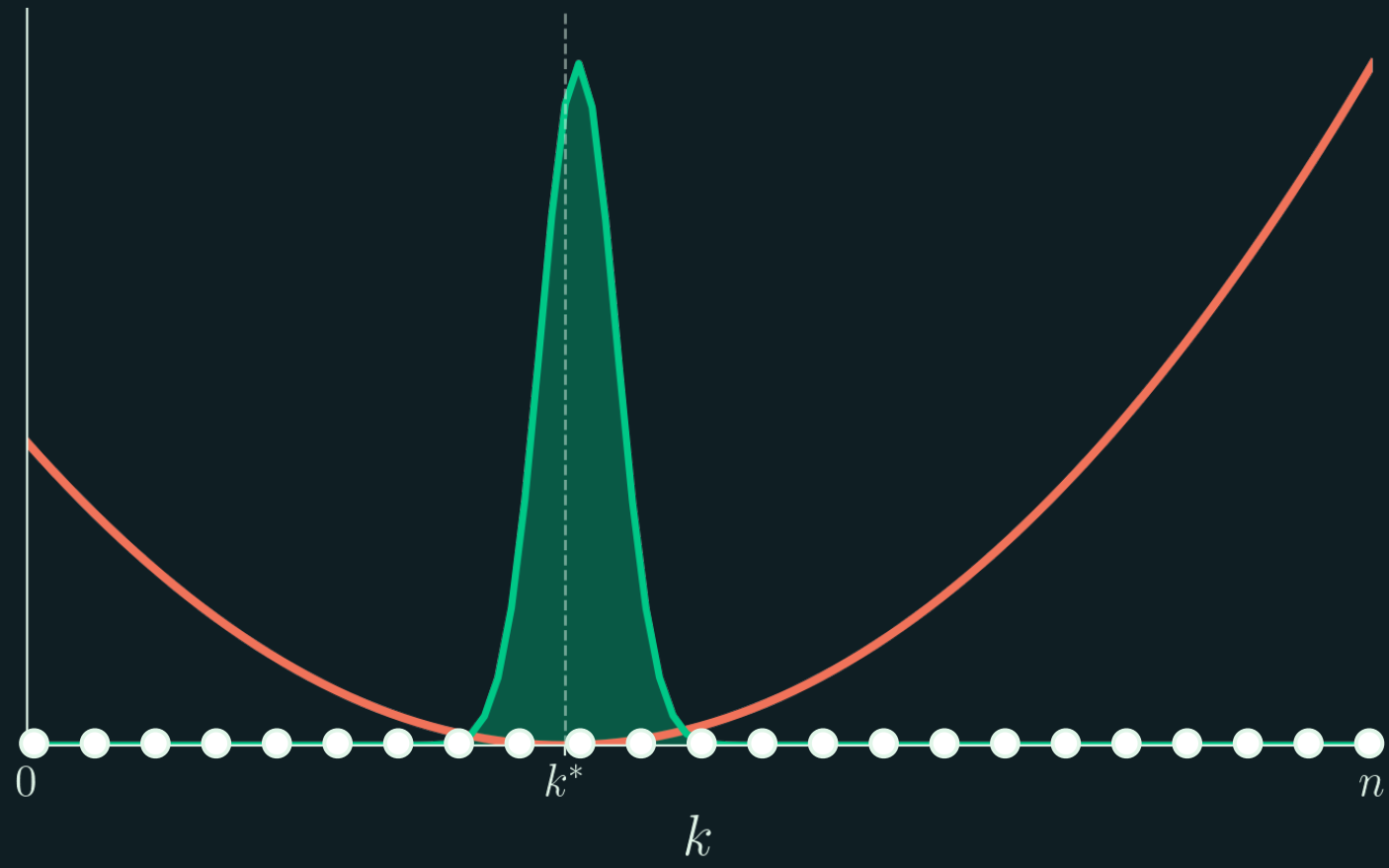
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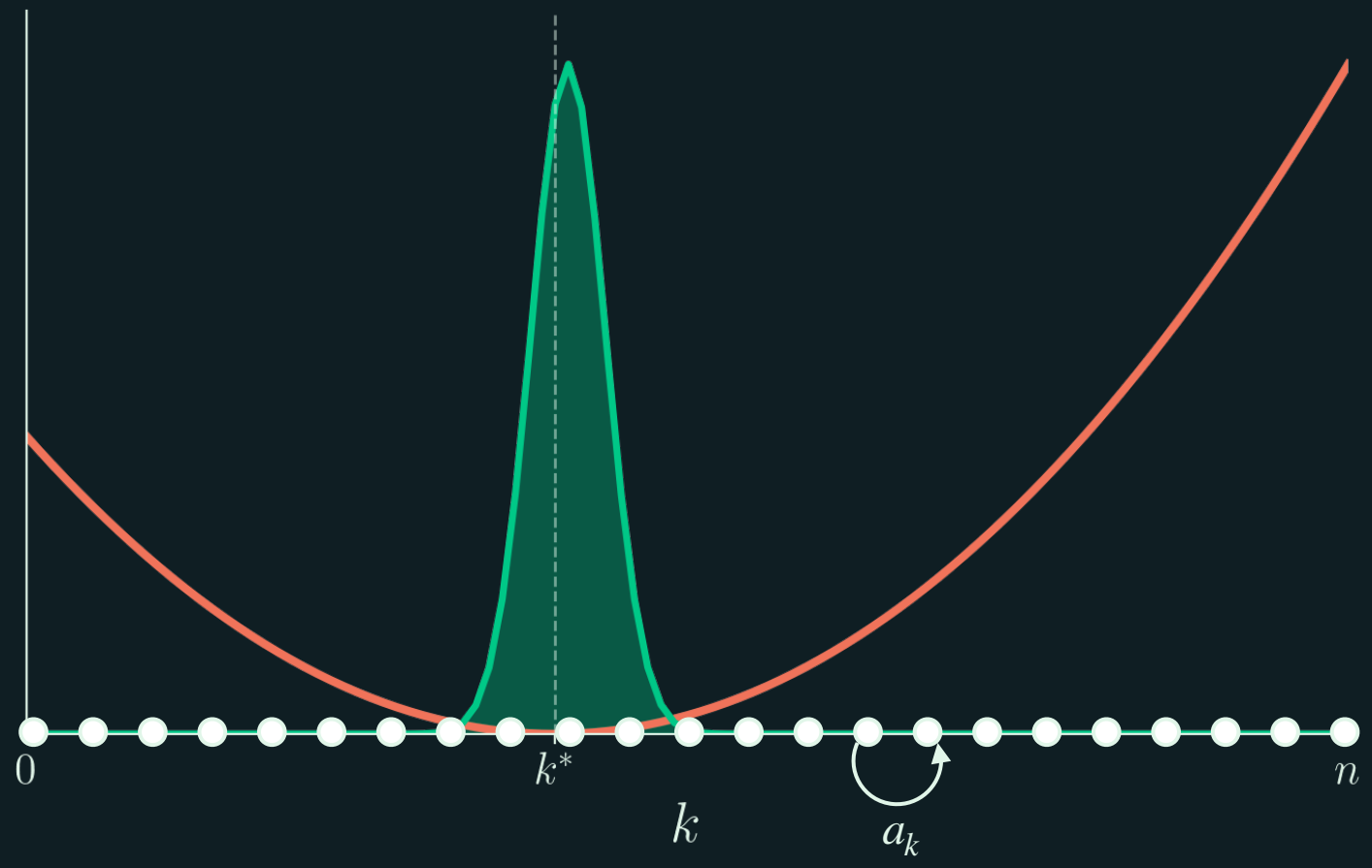
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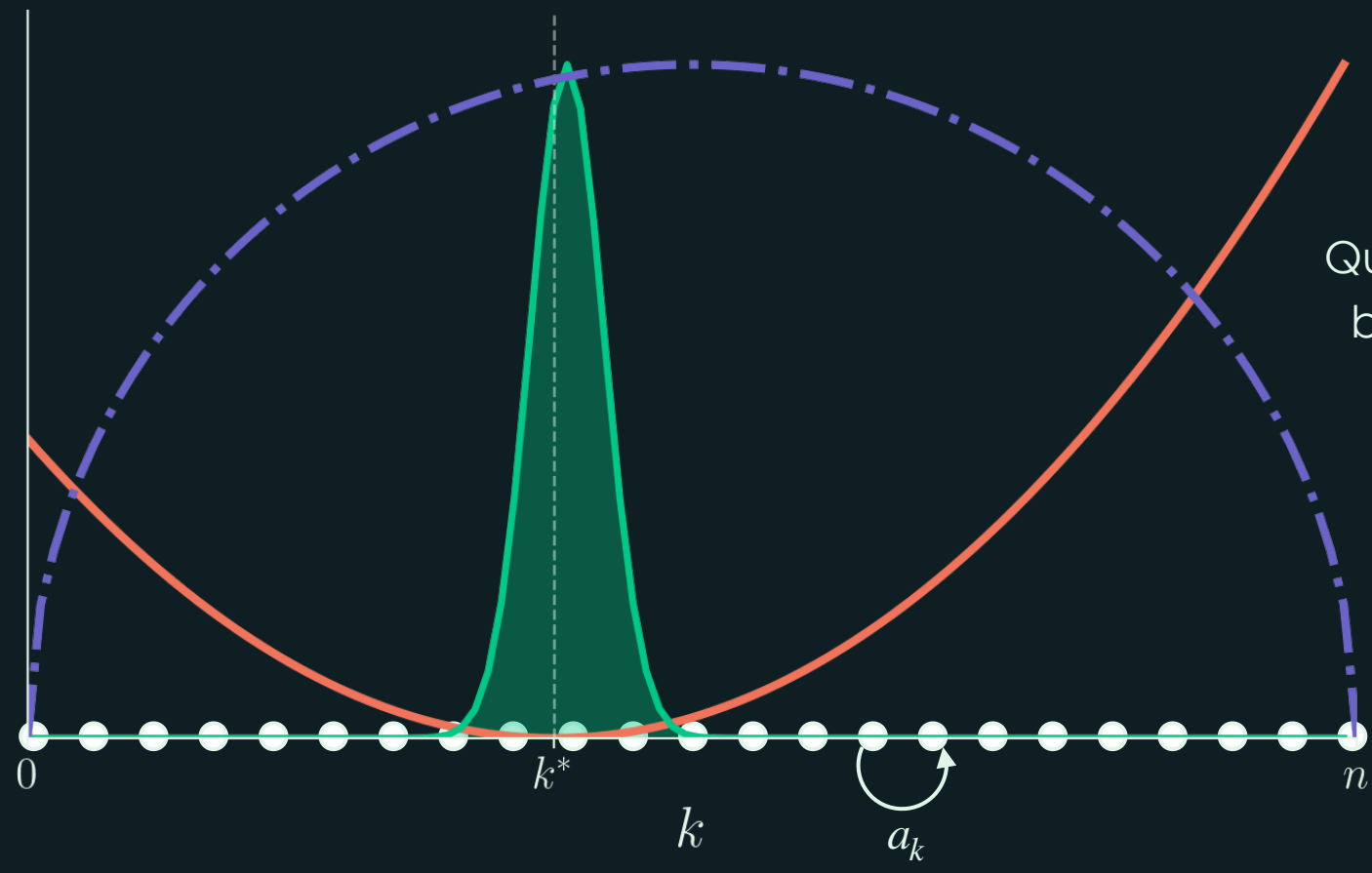
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Quantum: Competition between potential V and mixing A

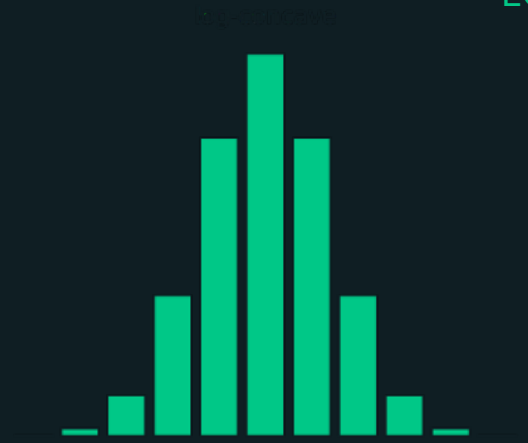
Log-concave distributions are everywhere

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Log concavity : $p_k^2 \geq p_{k+1} p_{k-1}$

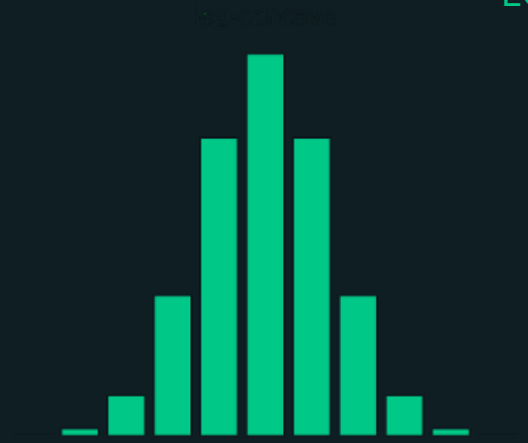
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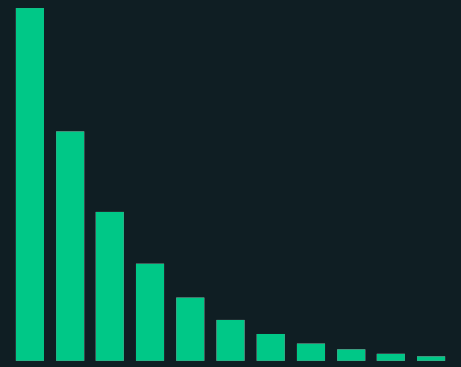
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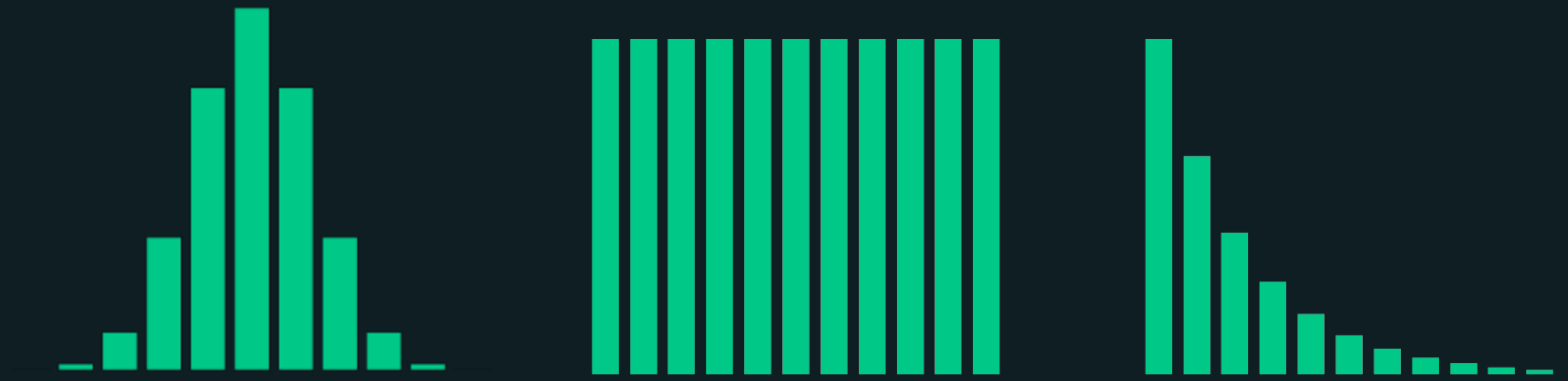
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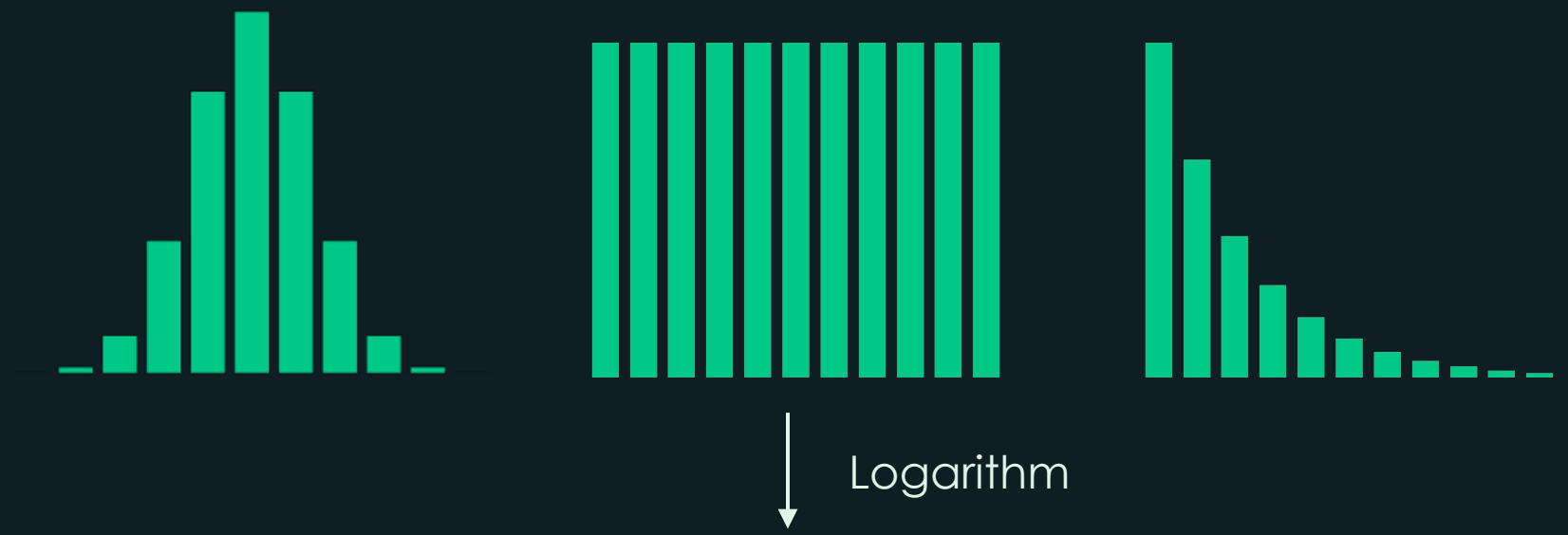
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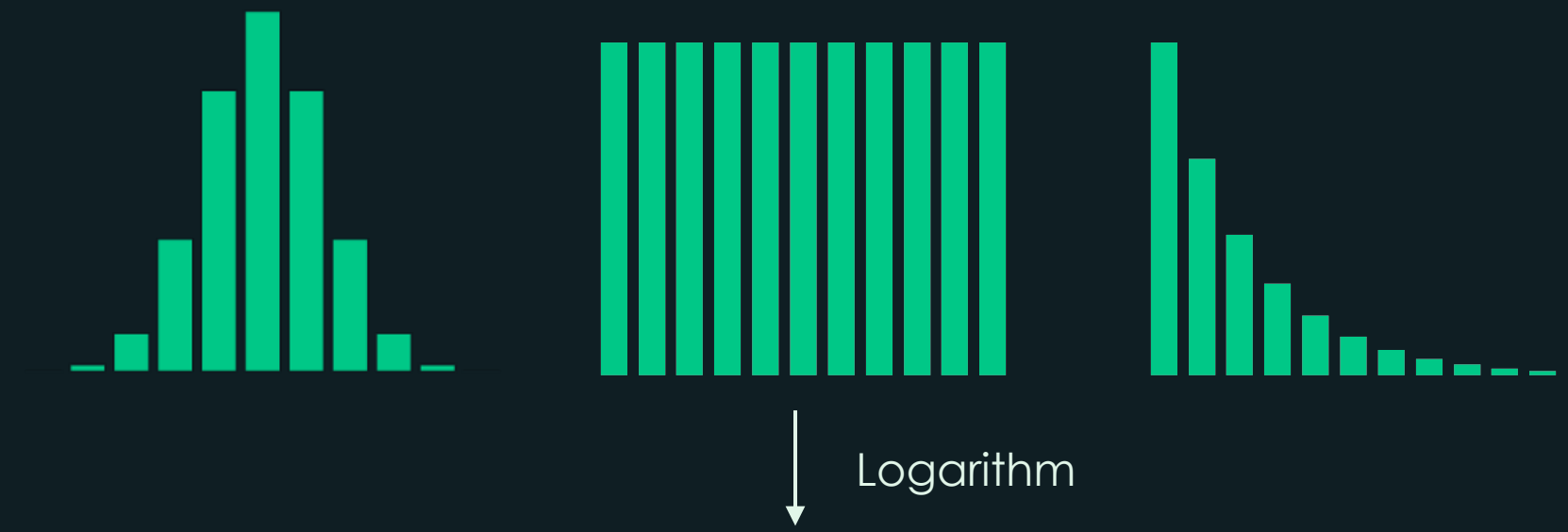
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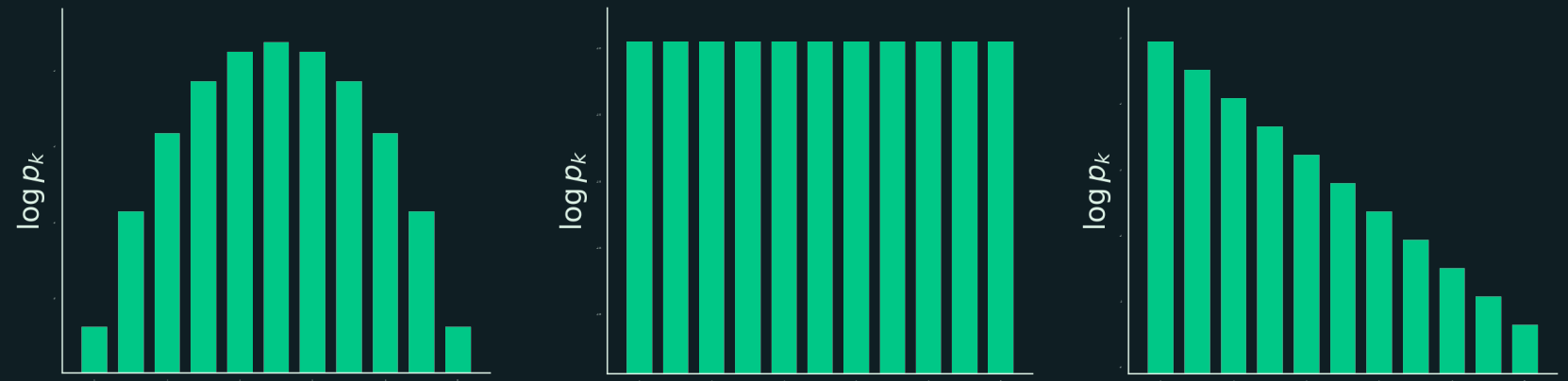
Logarithm

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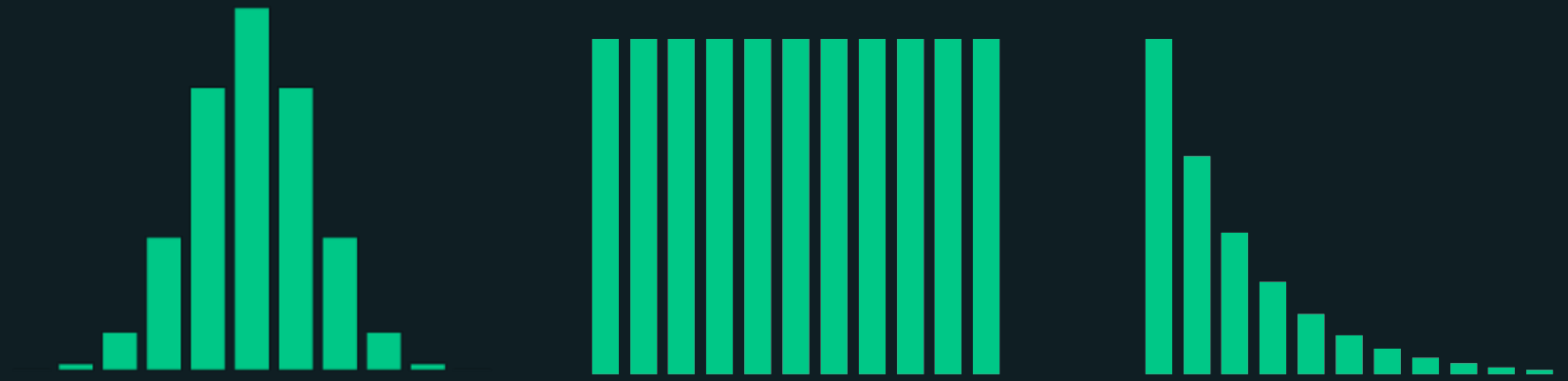


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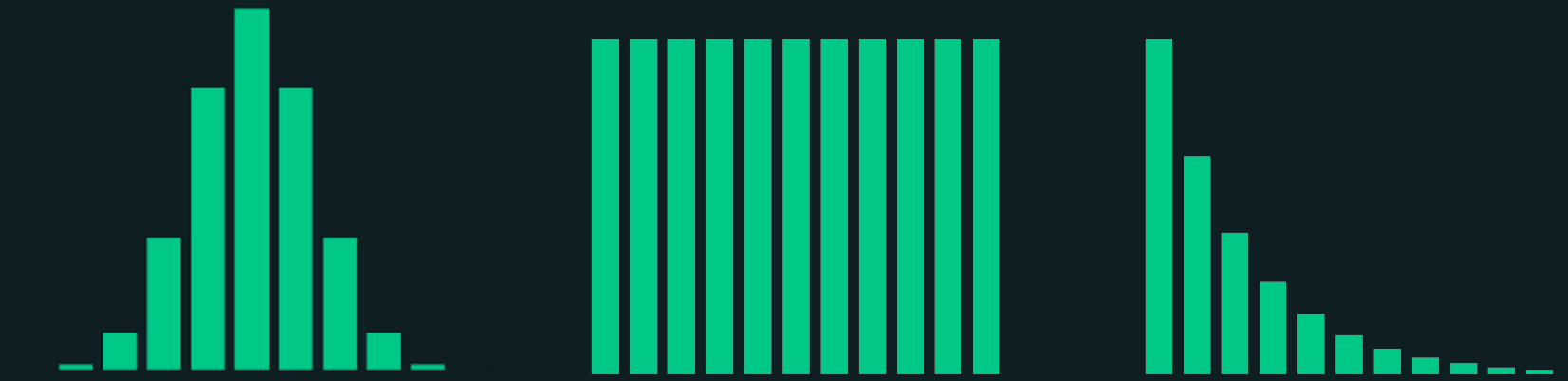
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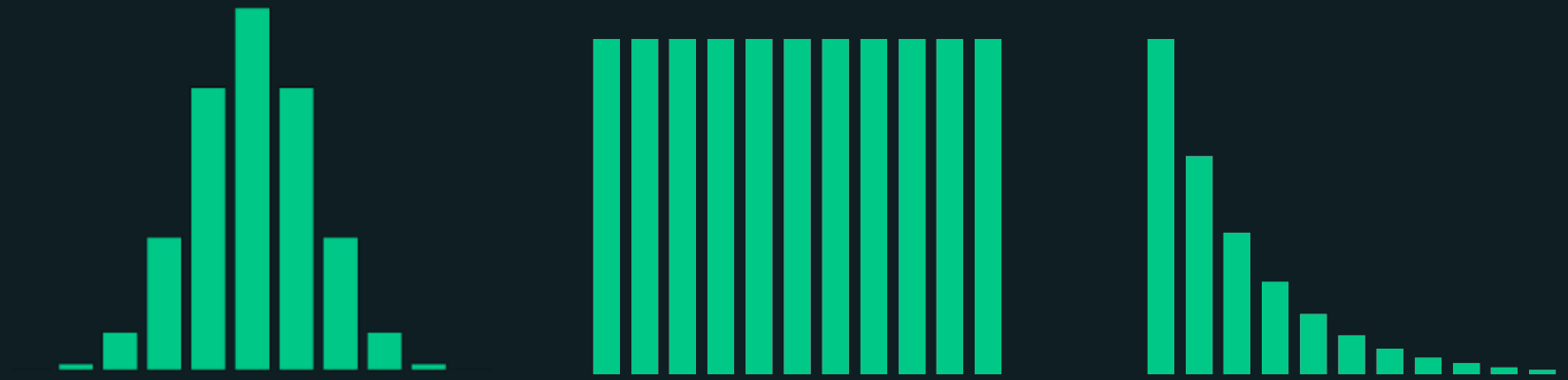
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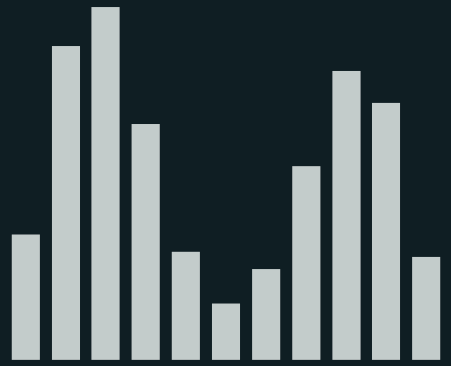
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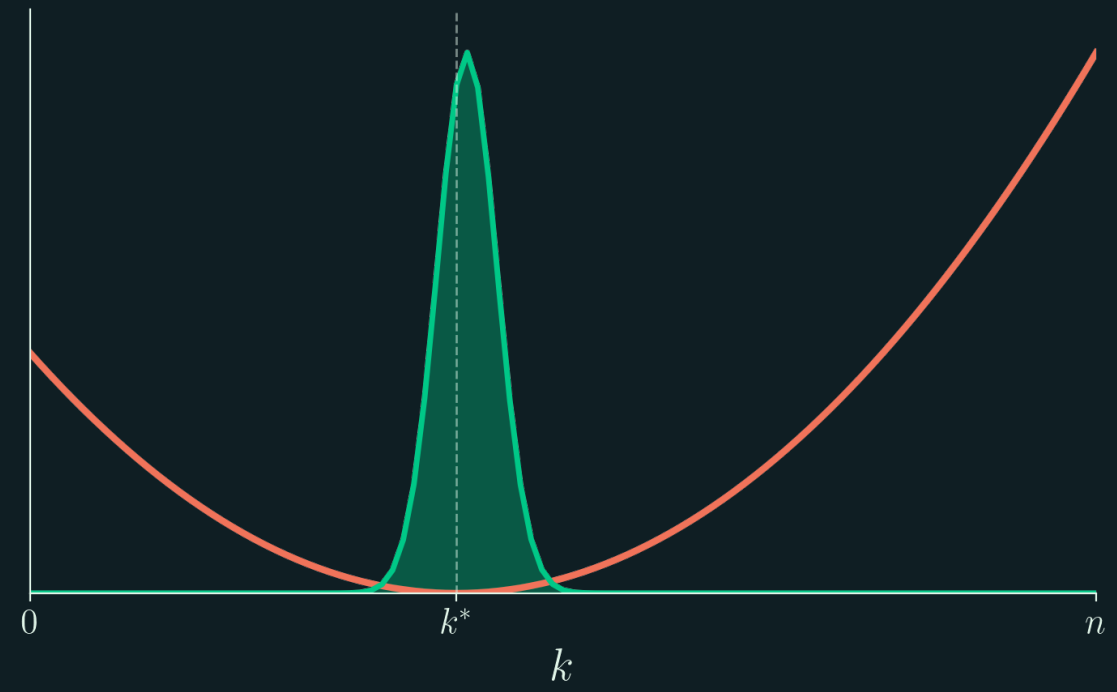
Convex optimization

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Continuous setting [Brascamp and Lieb 1972]

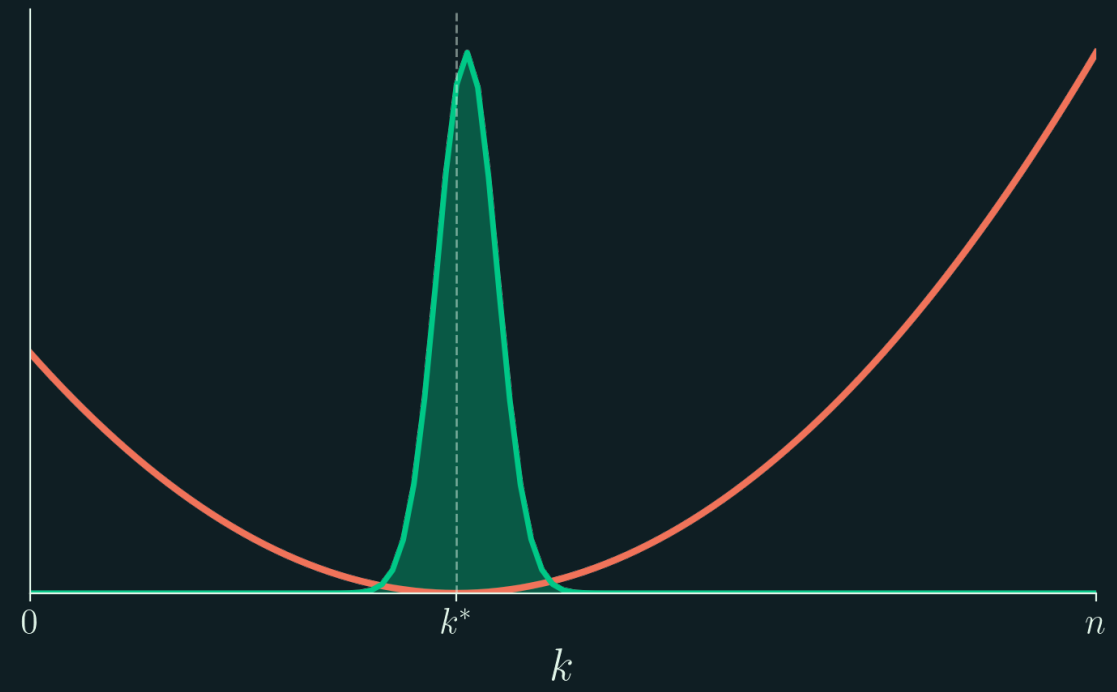
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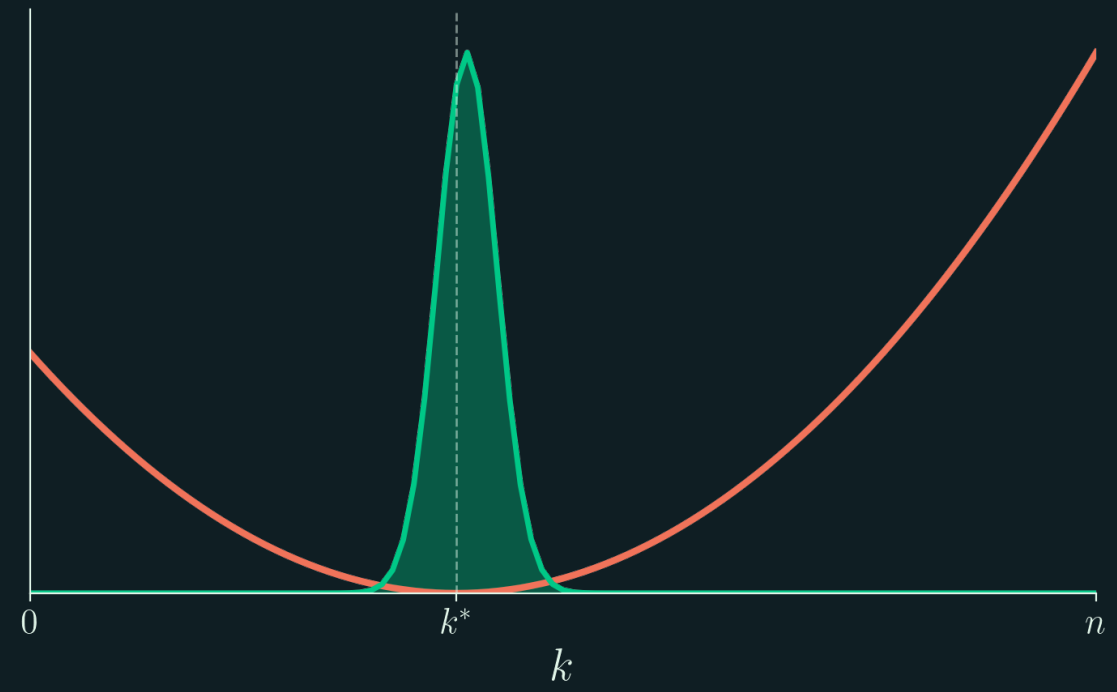
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Quadratic potential ⇒ Gaussian ground state

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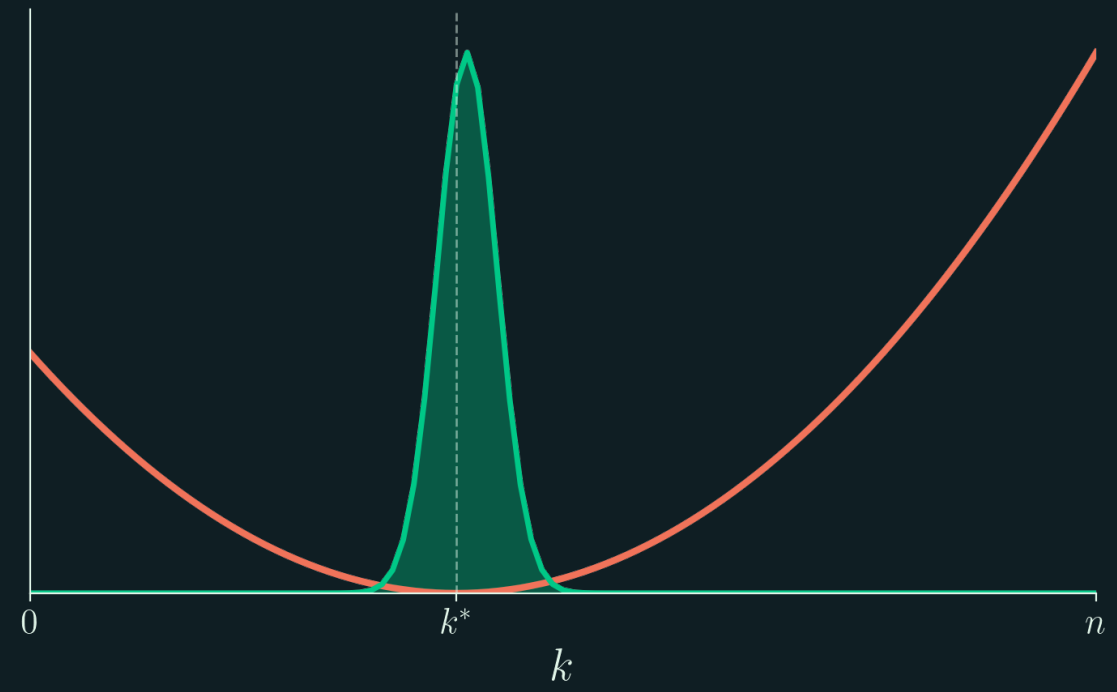


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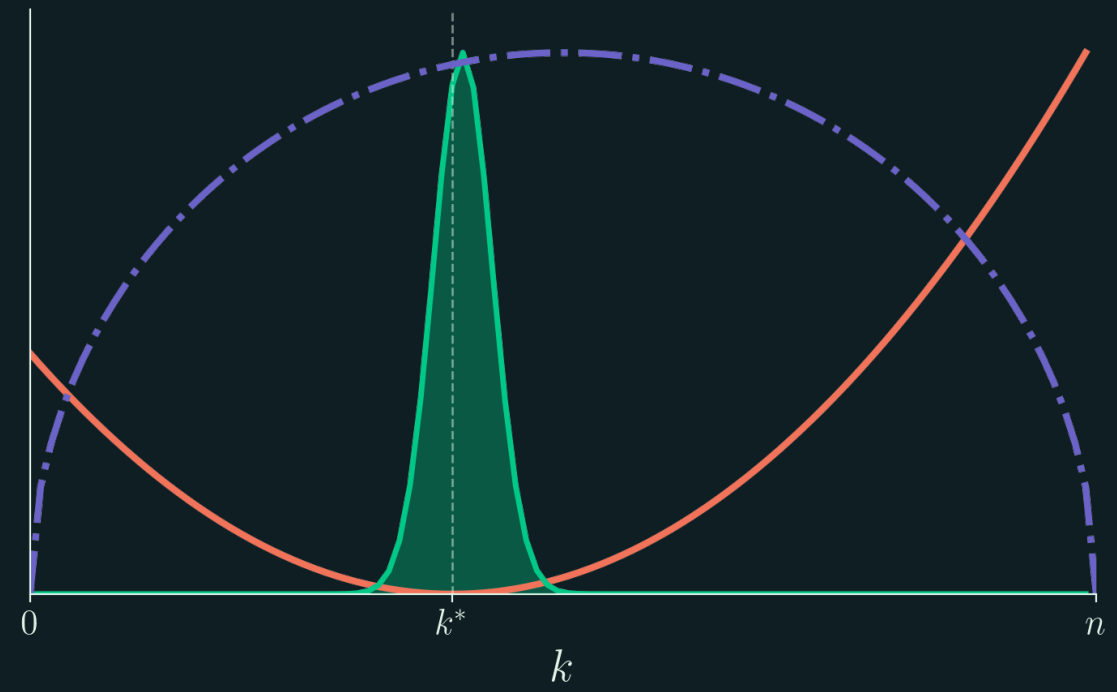
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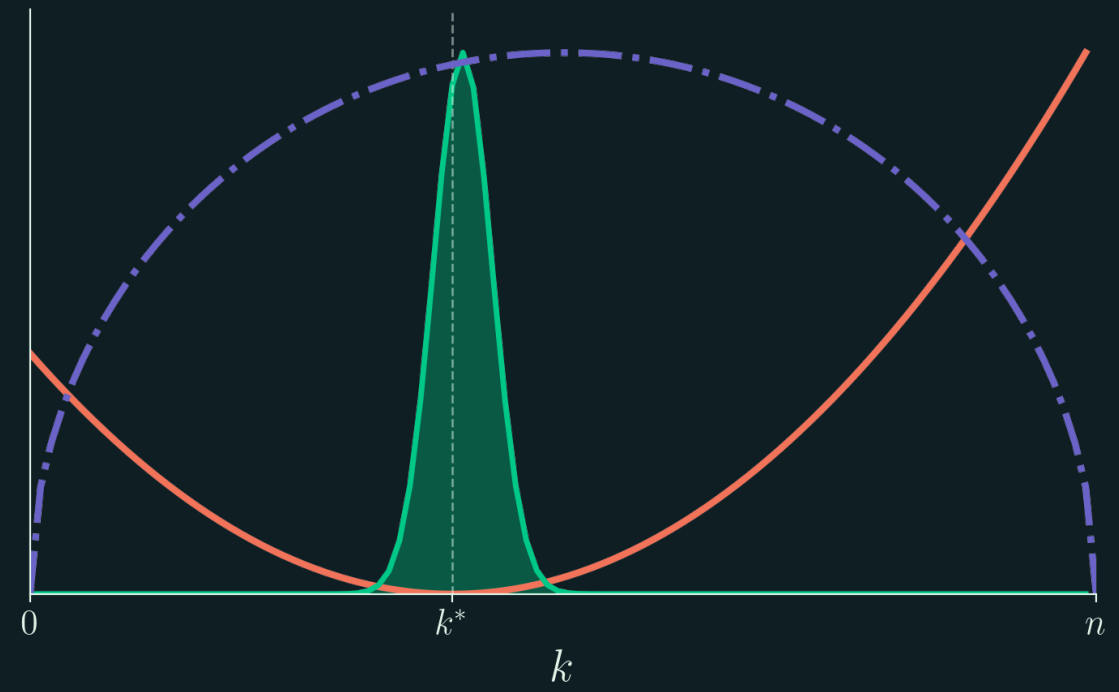
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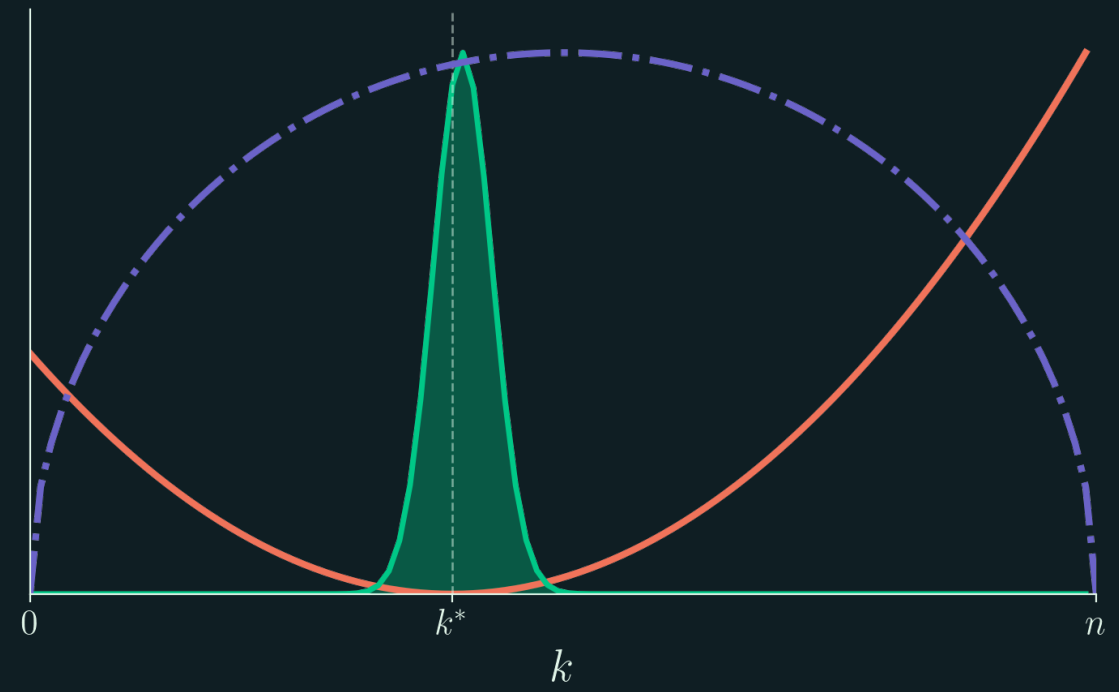
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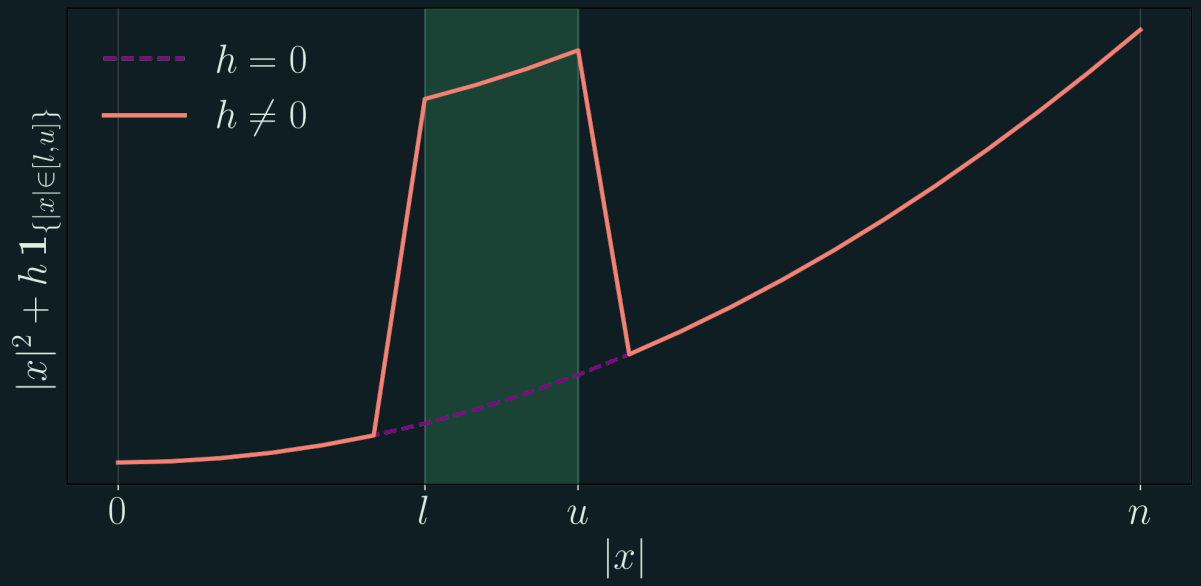
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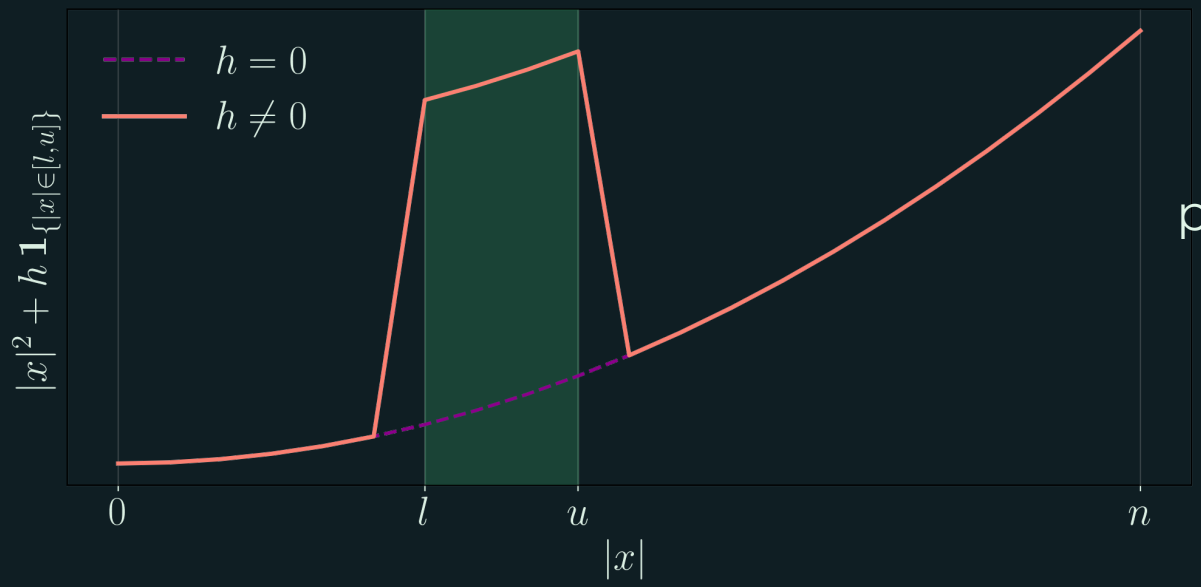
Δ is constant

Application: Convex with spike - Our contribution

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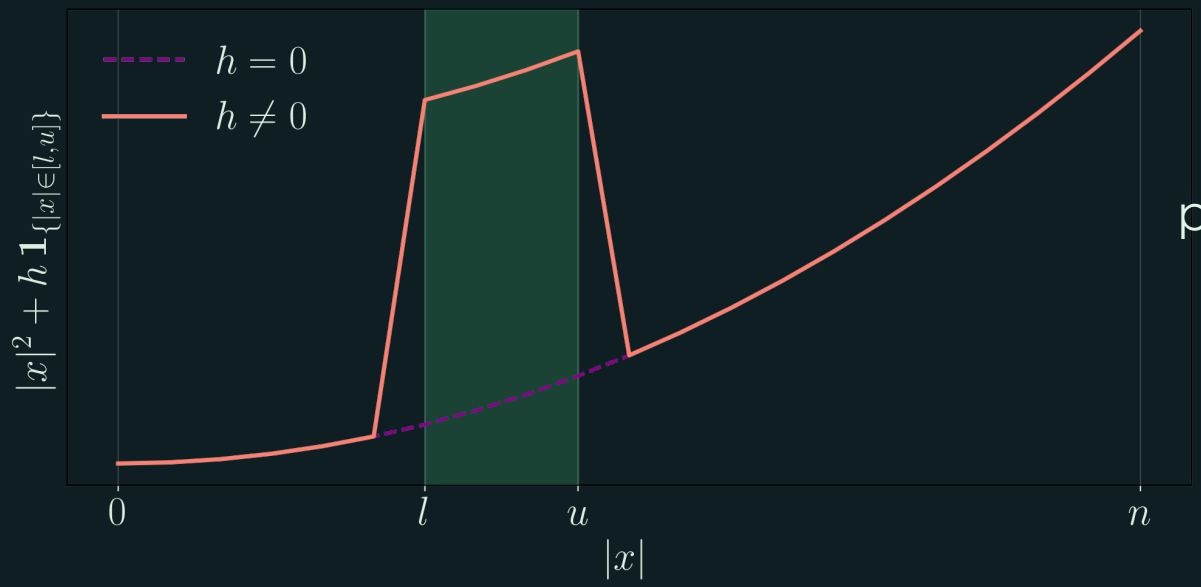
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Can the spike perturb the ground state?

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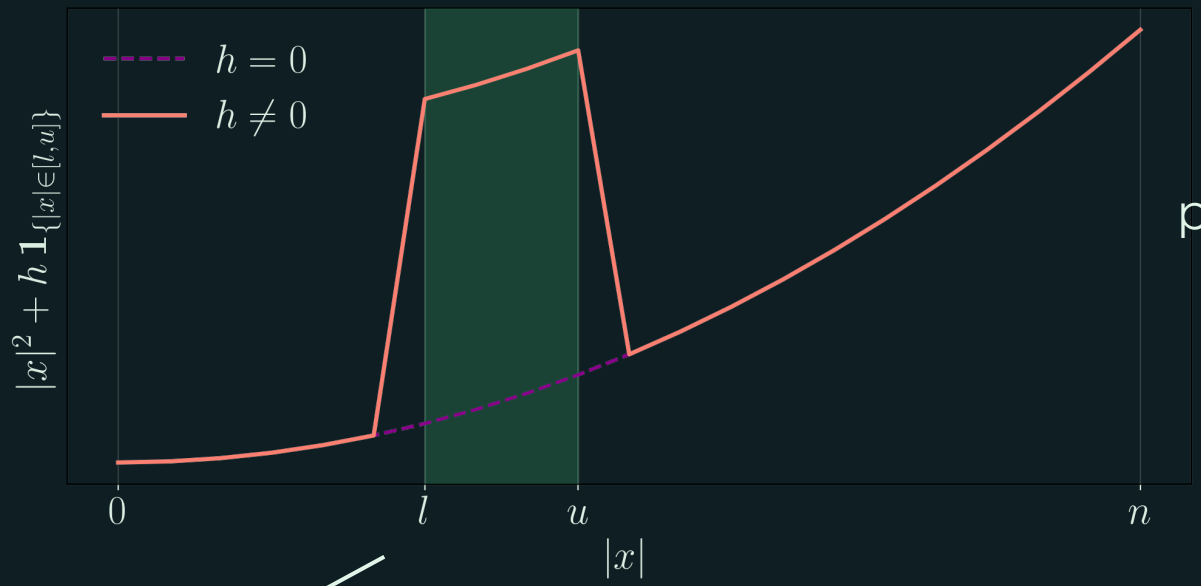
Log-concavity: unimodal + decay of amplitude away from the mode



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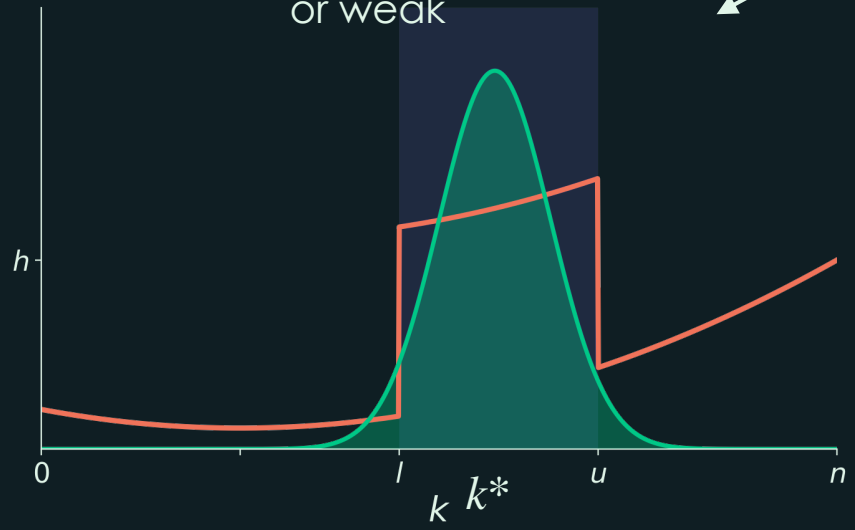
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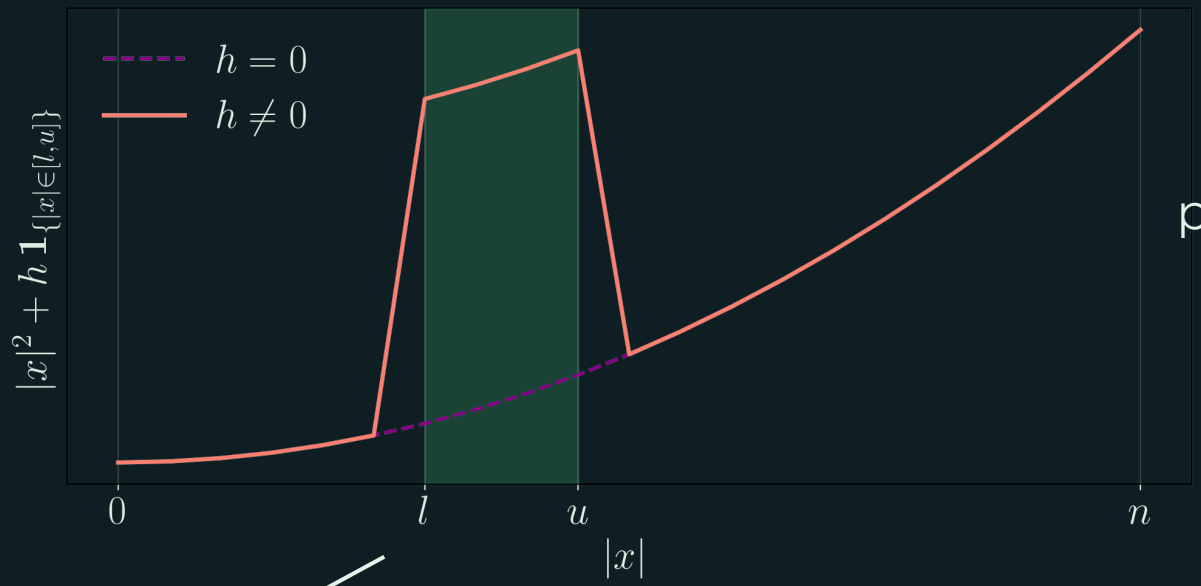
Can the spike perturb the ground state?

Close to the mode: Little effect if sufficiently narrow or weak



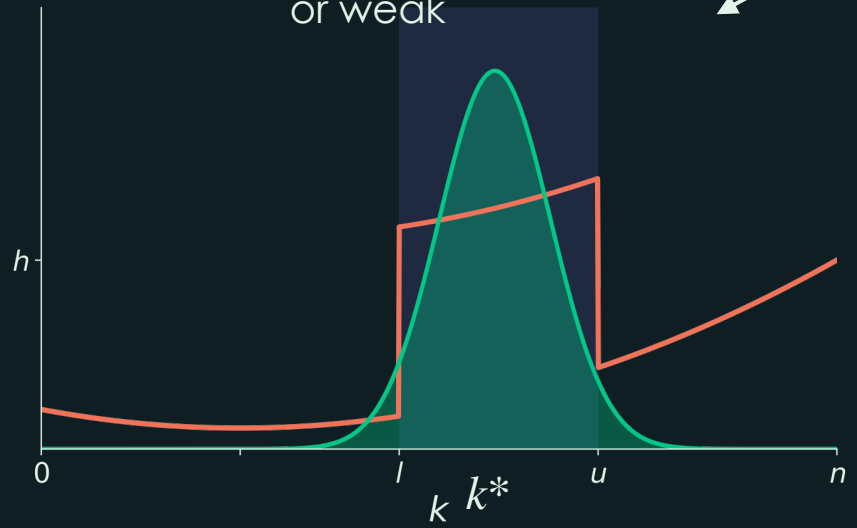
Application: Convex with spike - Our contribution

Log-concavity: unimodal + decay of amplitude away from the mode

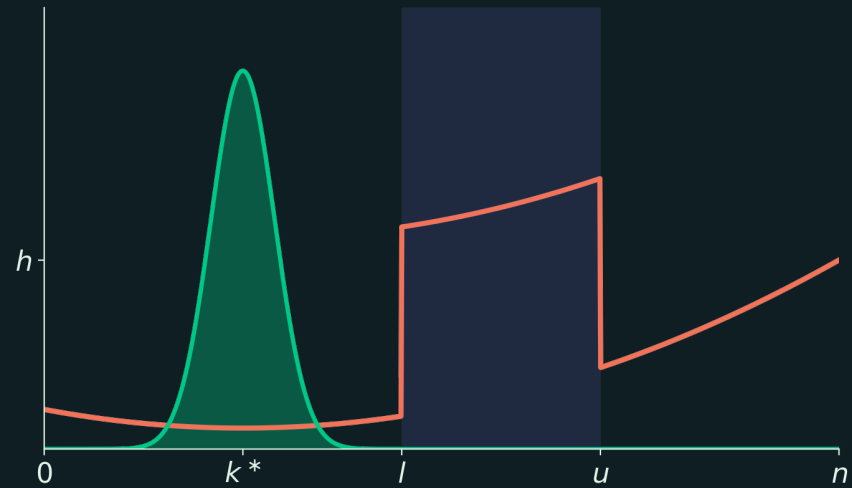


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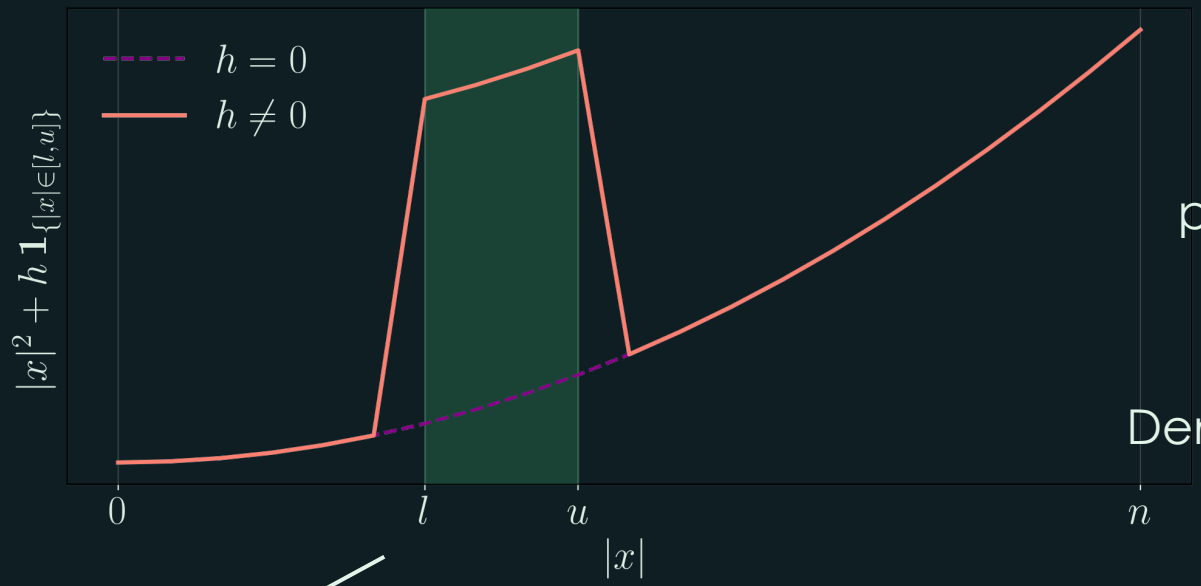


Away from the mode: Exponential decay tails



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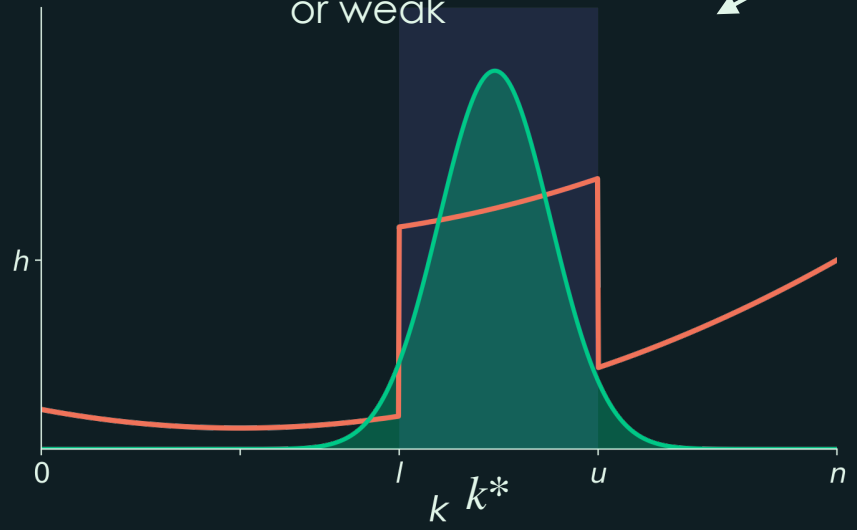
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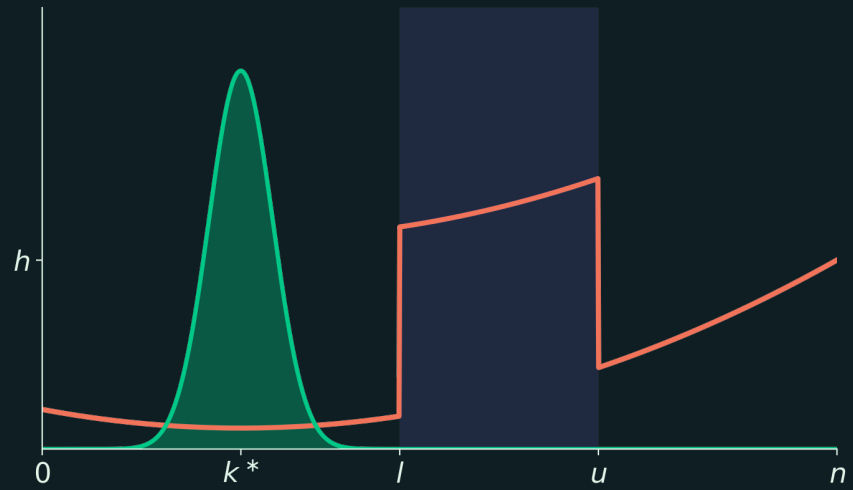
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Future work:
Density of IS ↔ Convex + spike

Close to the mode: Little effect if sufficiently narrow or weak



Away from the mode: Exponential decay tails



Can we do better than only convex? - Our contributions

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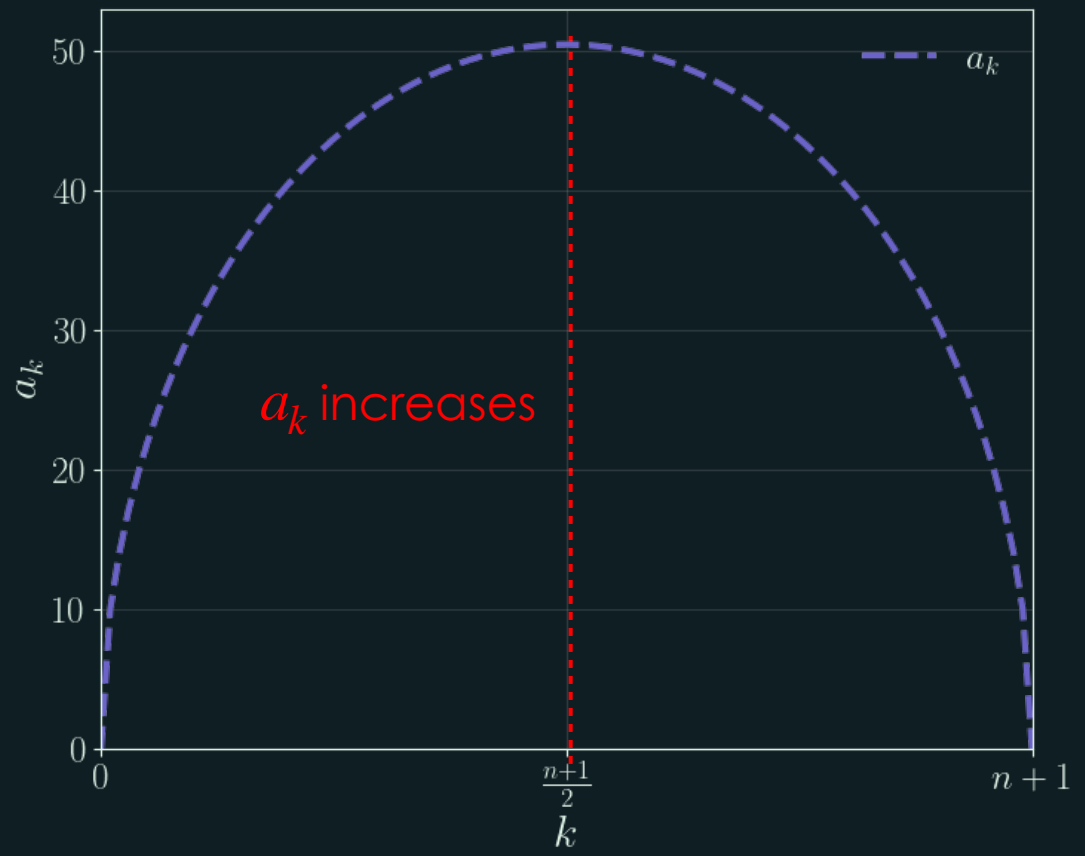
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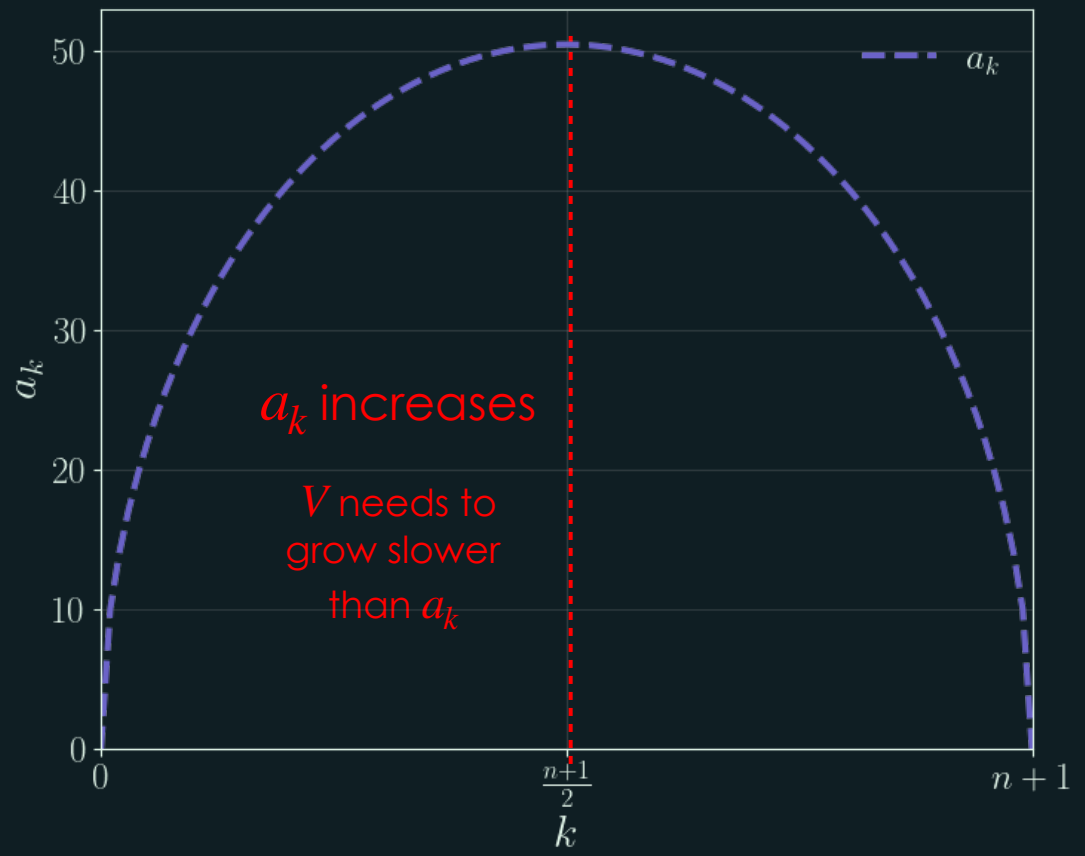
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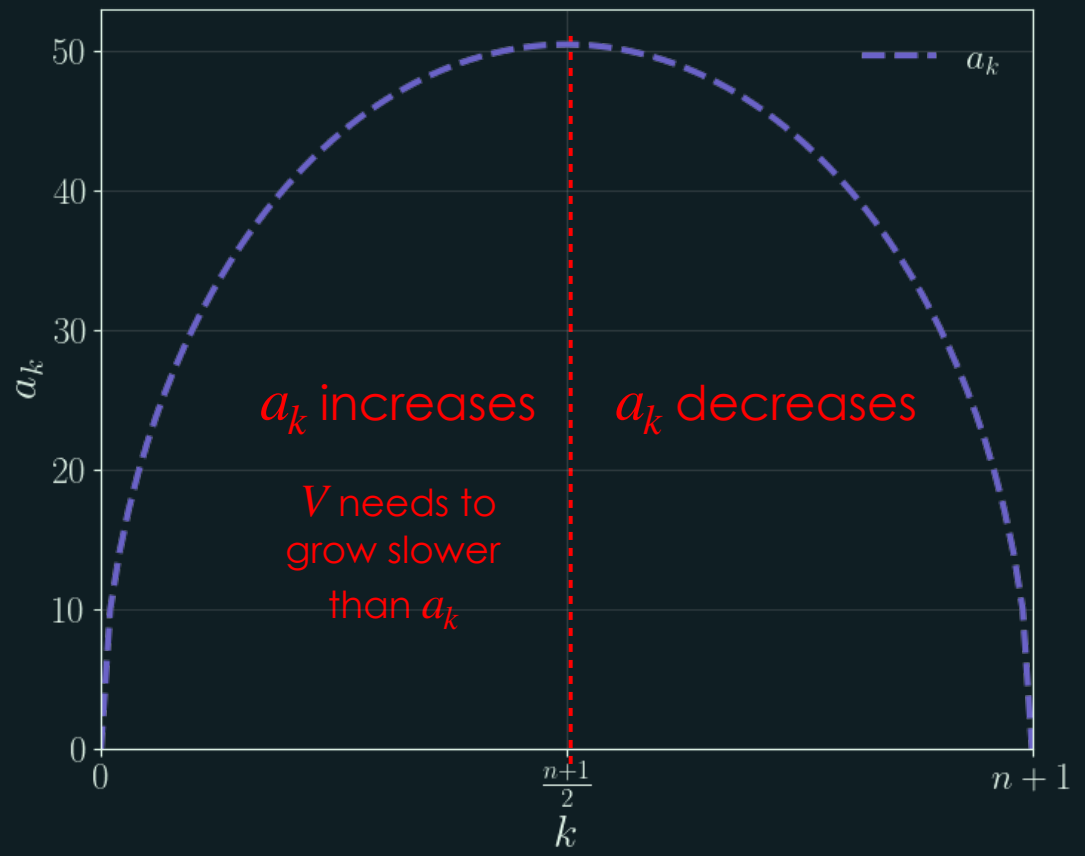
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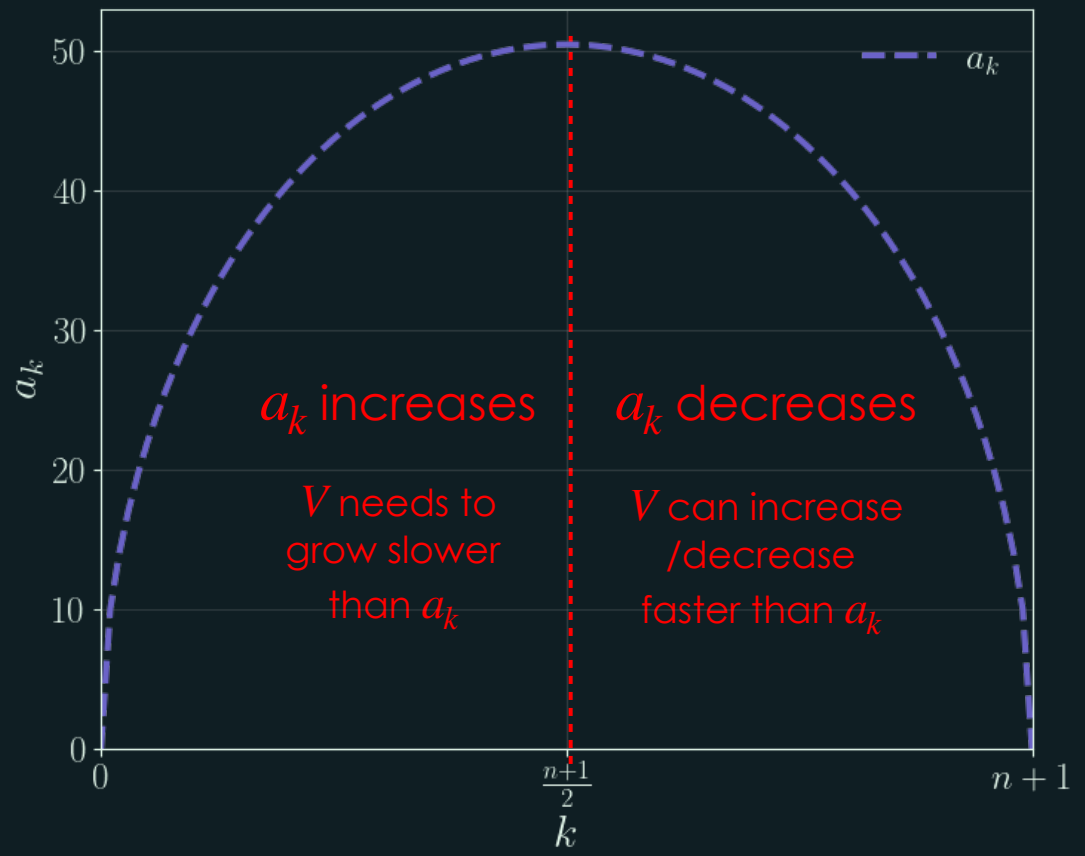
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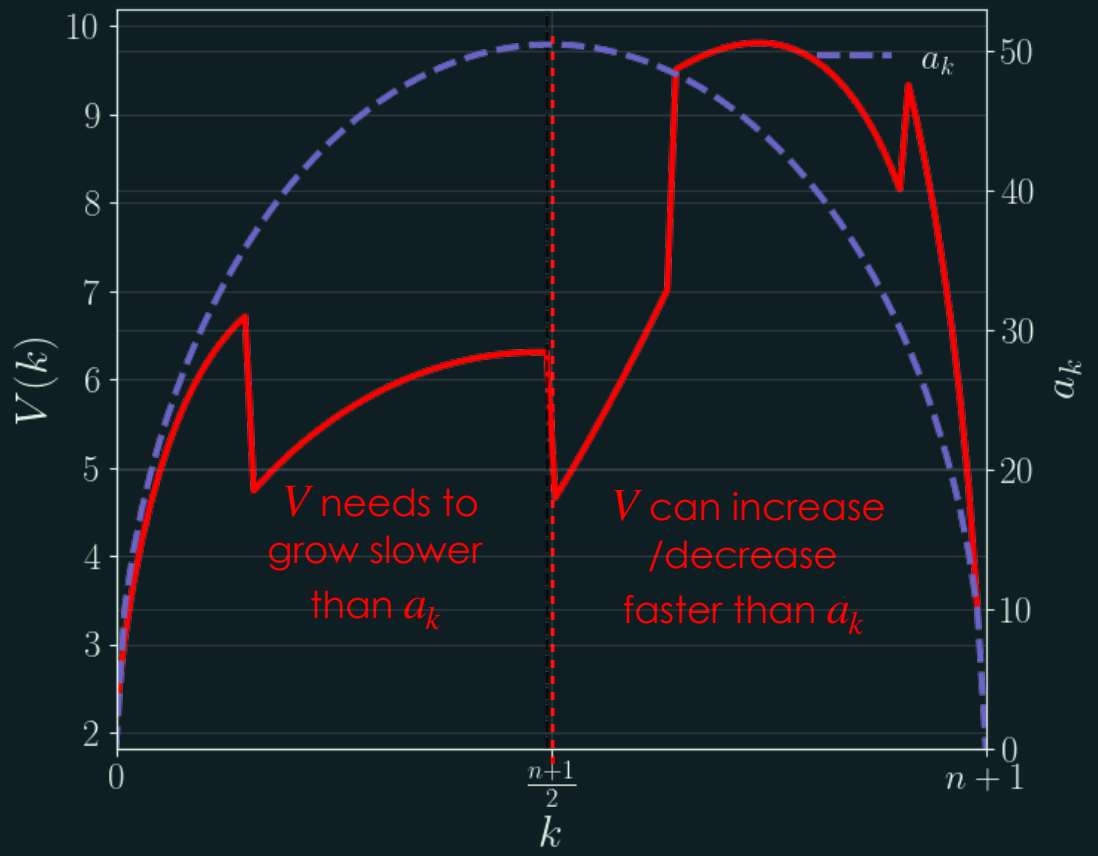
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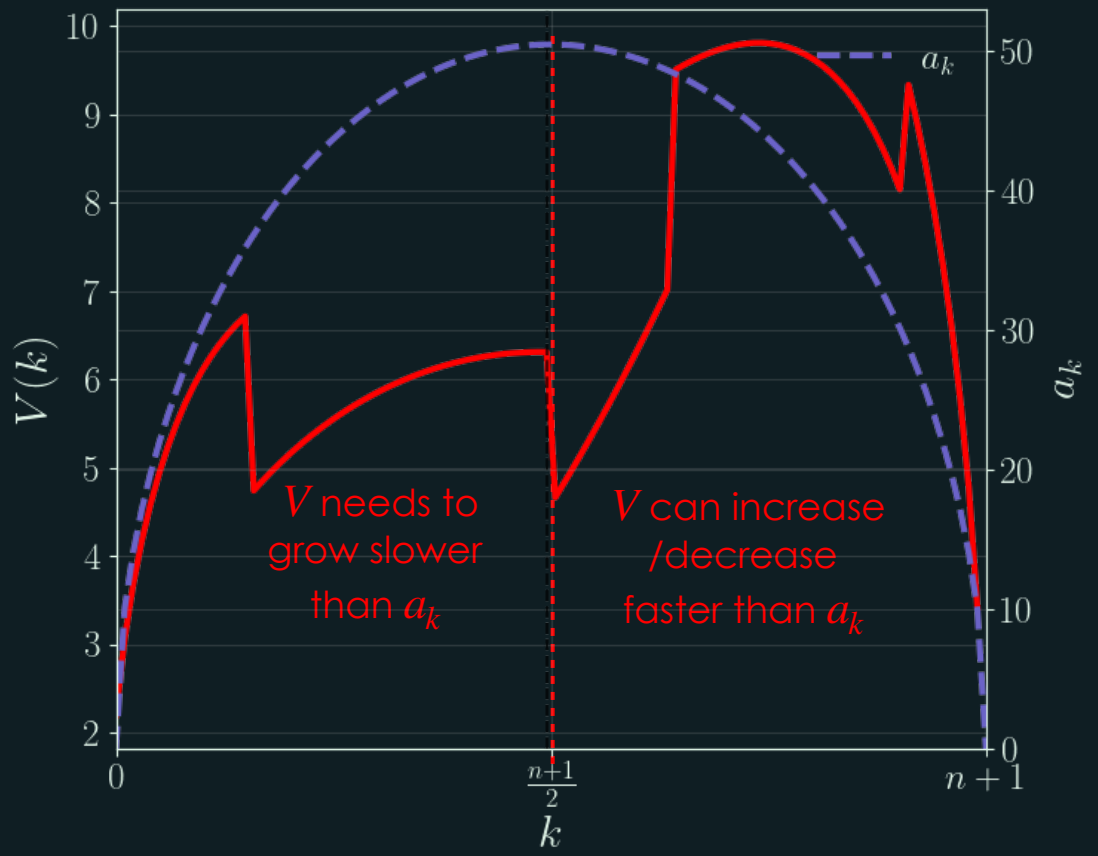
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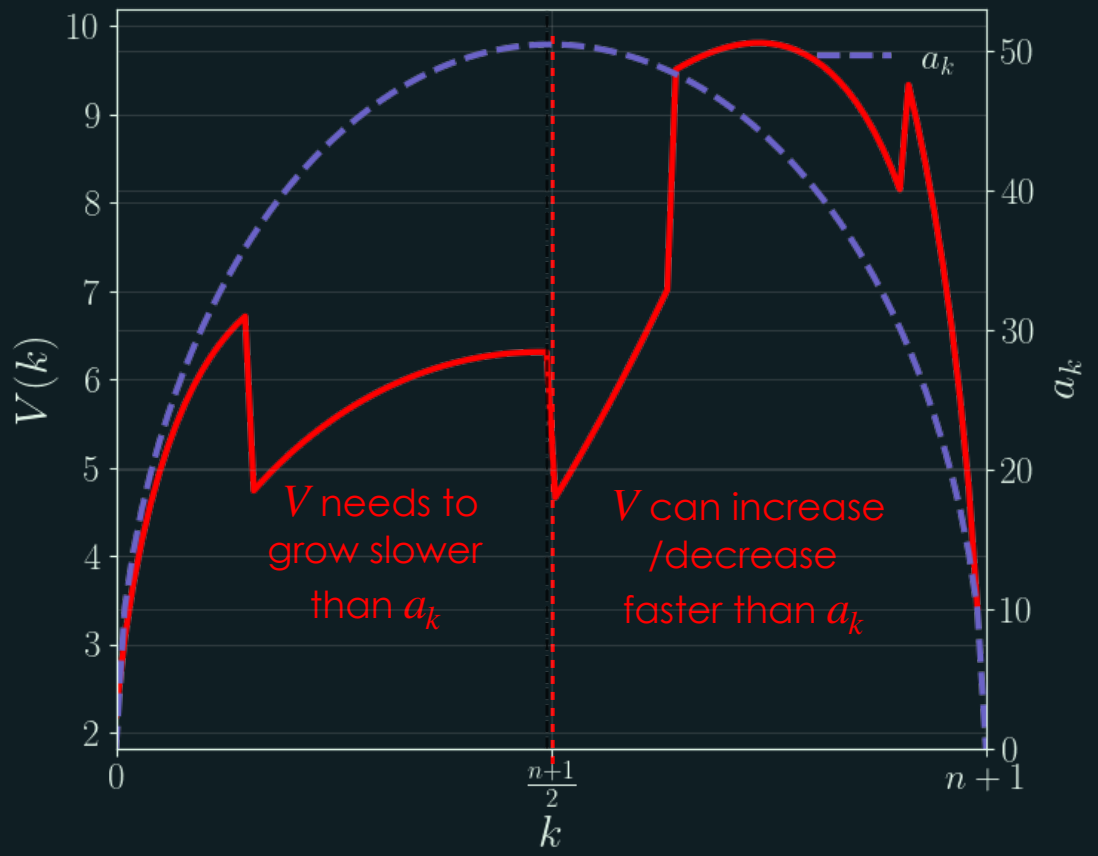
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The potential can be peaked!

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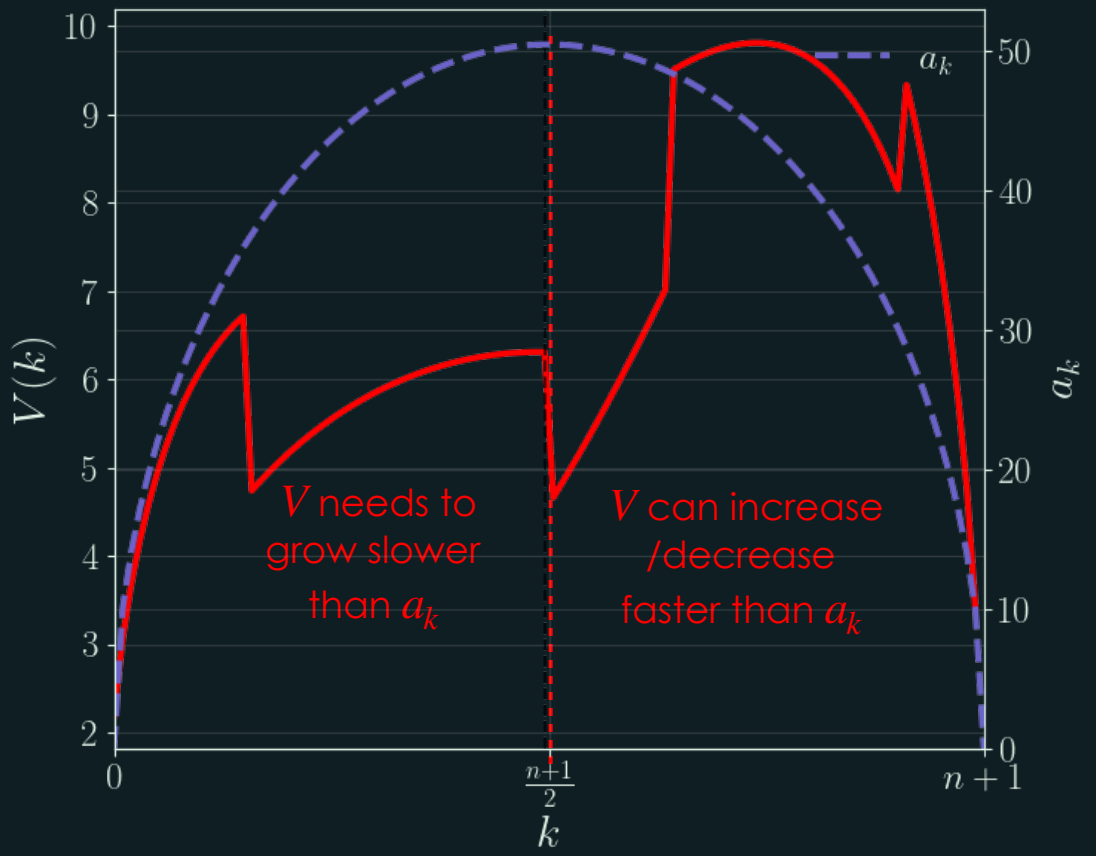
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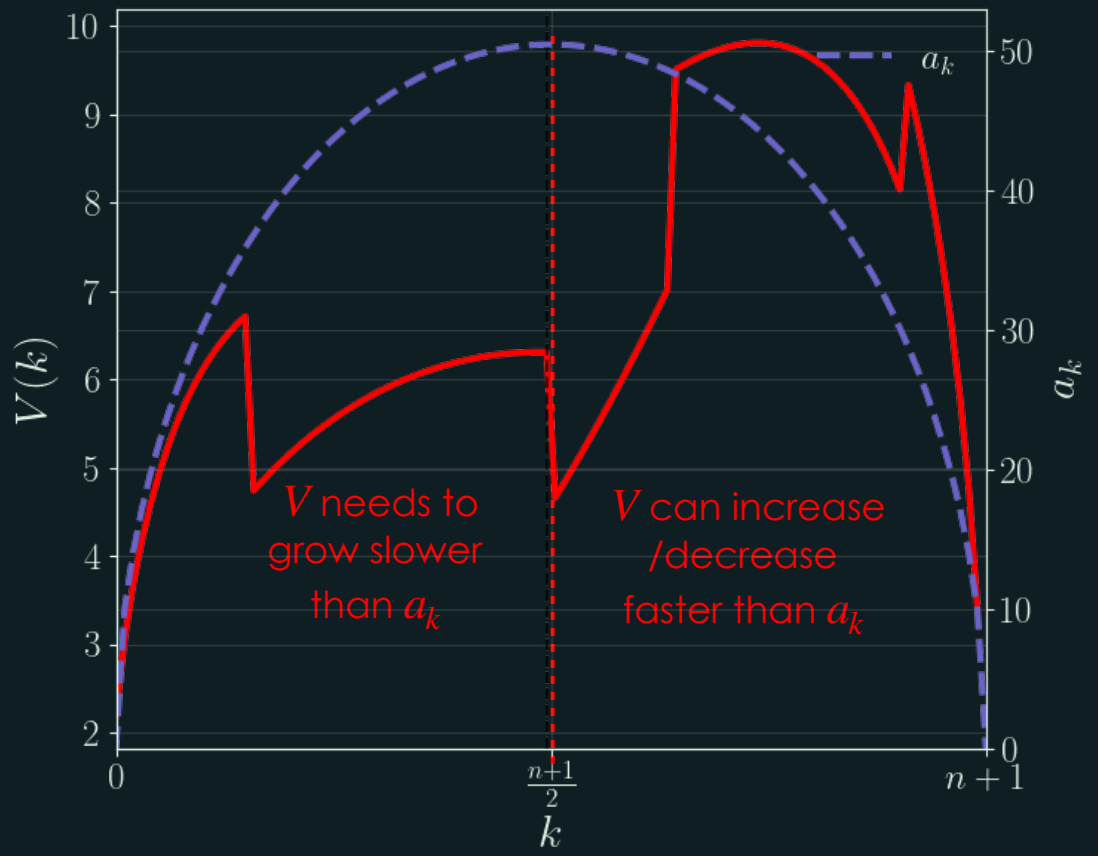
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Future work: can this be used to study the density of independent set when local minima?

Pasqal Defining the quantum reality

Conclusions and outlooks



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Thank you !